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# **$WZ$ Production at Hadron Colliders: Effects of Non-standard $WWZ$ Couplings and QCD Corrections**

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## **Abstract**

The process  $p\bar{p} \xrightarrow{(\cdot)} W^\pm Z + X \rightarrow \ell_1^\pm \nu_1 \ell_2^\pm \ell_2^- + X$  is calculated to  $\mathcal{O}(\alpha_s)$  for general  $C$  and  $P$  conserving  $WWZ$  couplings. At the Tevatron center of mass energy, the QCD corrections to  $WZ$  production are modest. At Large Hadron Collider (LHC) energies, the inclusive QCD corrections are large, but can be reduced significantly if a jet veto is imposed. Sensitivity limits for the anomalous  $WWZ$  couplings are derived from the next-to-leading order  $Z$  boson transverse momentum distribution for Tevatron and LHC energies. Unless a jet veto is imposed, next-to-leading order QCD corrections decrease the sensitivity to anomalous  $WWZ$  couplings considerably at LHC energies, but have little influence at the Tevatron. We also study, at next-to-leading order, rapidity correlations between the  $W$  and  $Z$  decay products, and the  $ZZ/WZ$  and  $WZ/W\gamma$  cross section ratios. These quantities are found to be useful tools in searching for the approximate zero present in the Standard Model  $WZ$  helicity amplitudes. The prospects for observing the approximate amplitude zero at the Tevatron and the LHC are critically assessed.

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## I. INTRODUCTION

The electroweak Standard Model (SM) based on an  $SU(2) \otimes U(1)$  gauge theory has been remarkably successful in describing contemporary high energy physics experiments. The three vector boson couplings predicted by this non-abelian gauge theory, however, remain largely untested. The production of  $WZ$  pairs at hadron colliders provides an excellent opportunity to study the  $WWZ$  vertex [1,2,3,4,5]. In addition, the reaction  $p\bar{p} \rightarrow W^\pm Z$  is of interest due to the presence of an approximate zero in the amplitude of the parton level subprocess  $q_1\bar{q}_2 \rightarrow W^\pm Z$  [5] in the SM, which is similar in nature to the well-known radiation zero in the reaction  $p\bar{p} \rightarrow W^\pm\gamma$  [6]. In the SM, the  $WWZ$  vertex is completely fixed by the  $SU(2) \otimes U(1)$  gauge structure of the electroweak sector. A measurement of the  $WWZ$  vertex thus provides a stringent test of the SM.

$WZ$  production at hadron colliders has recently received attention due to the observation of a clean  $W^+Z \rightarrow e^+\nu_e e^+e^-$  candidate event by the CDF Collaboration [7]. Double leptonic  $WZ$  decays are relatively background free and therefore provide an excellent testing ground for anomalous  $WWZ$  couplings. With an integrated luminosity of the order of  $1 \text{ fb}^{-1}$ , which is envisioned for the Main Injector era [8], a sufficient number of events should be available to commence a detailed investigation of the  $WWZ$  vertex in the  $W^\pm Z \rightarrow \ell_1^\pm\nu_1\ell_2^+\ell_2^-$  channel ( $\ell_1, \ell_2 = e, \mu$ ). The prospects for a precise measurement of the  $WWZ$  couplings in this channel would further improve if integrated luminosities on the order of  $10 \text{ fb}^{-1}$  could be achieved (a luminosity upgraded Tevatron will henceforth be denoted by  $\text{TeV}^*$ ) and/or the energy of the Tevatron could be doubled to  $\sqrt{s} = 3.5 - 4 \text{ TeV}$  (an energy upgraded Tevatron will henceforth be referred to as the DiTevatron) [8]. At the Large Hadron Collider (LHC,  $pp$  collisions at  $\sqrt{s} = 14 \text{ TeV}$  [9]), it should be possible to determine the  $WWZ$  couplings with high precision [3].

In contrast to low energy data and high precision measurements at the  $Z$  peak, collider experiments offer the possibility of a direct, and essentially model independent, determination of the three vector boson vertices. Hadronic production of  $WZ$  pairs was first calculated

in Ref. [1]. The  $\mathcal{O}(\alpha_s)$  QCD corrections to the reaction  $p\bar{p} \rightarrow W^\pm Z$  were first evaluated in Refs. [10] and [11]. Studies on the potential for probing the  $WWZ$  vertex have been performed in Refs. [3] and [4].

Previous studies on probing the  $WWZ$  vertex via hadronic  $WZ$  production have been based on leading-order (LO) calculations [3,4]. In general, the inclusion of anomalous couplings at the  $WWZ$  vertex yields enhancements in the  $WZ$  cross section, especially at large values of the  $W$  or  $Z$  boson transverse momentum,  $p_T(W)$  or  $p_T(Z)$ , and at large values of the  $WZ$  invariant mass,  $M_{WZ}$ . Next-to-leading-order (NLO) calculations of hadronic  $WZ$  production have shown that the  $\mathcal{O}(\alpha_s)$  corrections are large in precisely these same regions [10,11]. It is thus vital to include the NLO corrections when using hadronic  $WZ$  production to test the  $WWZ$  vertex for anomalous couplings.

In this paper, we calculate hadronic  $WZ$  production to  $\mathcal{O}(\alpha_s)$ , including the most general,  $C$  and  $P$  conserving, anomalous  $WWZ$  couplings. Our calculation also includes the leptonic decays of the  $W$  and  $Z$  bosons in the narrow width approximation. Decay channels where the  $W$  or  $Z$  boson decays hadronically are not considered here. The calculation has been performed using the Monte Carlo method for NLO calculations [12]. With this method, it is easy to calculate a variety of observables simultaneously and to implement experimental acceptance cuts in the calculation. It is also possible to compute the NLO QCD corrections for exclusive channels, *e.g.*,  $p\bar{p} \rightarrow WZ + 0$  jet. Apart from anomalous contributions to the  $WWZ$  vertex we assume the SM to be valid in our calculation. In particular, we assume the couplings of the  $W$  and  $Z$  bosons to quarks and leptons are as given by the SM. Section II briefly summarizes the technical details of our calculation.

The results of our numerical simulations are presented in Secs. III and IV. In contrast to the SM contributions to the  $q_1\bar{q}_2 \rightarrow WZ$  helicity amplitudes, terms associated with non-standard  $WWZ$  couplings grow with energy. The  $WZ$  invariant mass distribution, the cluster transverse mass distribution, and the  $Z$  boson transverse momentum spectrum are therefore very sensitive to anomalous  $WWZ$  couplings. In Sec. III, we focus on the impact

QCD corrections have on these distributions, and the sensitivity limits for the anomalous  $WWZ$  couplings which can be achieved at the Tevatron, DiTevatron, and LHC with various integrated luminosities. At LHC energies, the inclusive NLO QCD corrections in the SM are found to be very large at high  $p_T(W)$  or  $p_T(Z)$ , and have a non-negligible influence on the sensitivity bounds which can be achieved for anomalous  $WWZ$  couplings. The large QCD corrections are caused by the combined effects of destructive interference in the Born subprocess, a log-squared enhancement factor in the  $q_1g \rightarrow WZq_2$  partonic cross section at high transverse momentum [11], and the large quark-gluon luminosity at supercollider energies. At the Tevatron, on the other hand, the  $\mathcal{O}(\alpha_s)$  QCD corrections are modest and sensitivities are only slightly affected by the QCD corrections. In Sec. III, we also show that the size of the QCD corrections at high  $p_T(Z)$  or  $p_T(W)$  can be significantly reduced, and a significant fraction of the sensitivity lost at LHC energies can be regained, if a jet veto is imposed, *i.e.*, if the  $WZ + 0$  jet exclusive channel is considered. We also find that the residual dependence of the NLO  $WZ + 0$  jet cross section on the factorization scale  $Q^2$  is significantly smaller than that of the  $\mathcal{O}(\alpha_s)$  cross section for the inclusive reaction  $p\bar{p} \rightarrow WZ + X$ .

In Sec. IV, we study rapidity correlations between the  $W$  and  $Z$  decay products, the  $ZZ/WZ$  and  $WZ/W\gamma$  cross section ratios, and how these quantities are affected by QCD corrections. In the SM at tree level, the process  $q_1\bar{q}_2 \rightarrow WZ$  exhibits an approximate amplitude zero at  $\cos\Theta^* \approx \frac{1}{3}\tan^2\theta_W \approx 0.1$  ( $\cos\Theta^* \approx -\frac{1}{3}\tan^2\theta_W \approx -0.1$ ) for  $u\bar{d} \rightarrow W^+Z$  ( $d\bar{u} \rightarrow W^-Z$ ) [5] which is similar in nature to the well-known radiation zero in  $W\gamma$  production in hadronic collisions. Here  $\Theta^*$  is the  $Z$  boson scattering angle in the parton center of mass frame relative to the quark direction and  $\theta_W$  is the weak mixing angle. The radiation zero in  $W\gamma$  production can be observed rather easily at the Tevatron in the distribution of the rapidity difference  $\Delta y(\gamma, \ell) = y(\gamma) - y(\ell)$  of the photon and the charged lepton which originates from the  $W$  decay,  $W \rightarrow \ell\nu$  [13]. In the SM, the  $\Delta y(\gamma, \ell)$  distribution exhibits a pronounced dip at  $\Delta y(\gamma, \ell) \approx \mp 0.3$  in  $W^\pm\gamma$  production, which originates from the radiation zero. We find that, at leading order, the approximate amplitude zero in

$p\bar{p} \rightarrow W^\pm Z \rightarrow \ell_1^\pm \nu_1 \ell_2^+ \ell_2^-$  leads to a dip in the corresponding  $\Delta y(Z, \ell_1) = y(Z) - y(\ell_1)$  distribution which is located at  $\Delta y(Z, \ell_1) \approx \pm 0.5$  ( $= 0$ ) in  $p\bar{p}$  ( $pp$ ) collisions. NLO QCD corrections tend to fill in the dip; at LHC energies they obscure the signal of the approximate amplitude zero almost completely, unless a jet veto is imposed.

Alternatively, cross section ratios can be considered. Analogous to the  $Z\gamma/W\gamma$  cross section ratio [14], the  $ZZ/WZ$  cross section ratio as a function of the minimum transverse momentum of the  $Z$  boson is found to reflect the approximate amplitude zero. The ratio of the  $WZ$  to  $W\gamma$  cross sections, on the other hand, measures the relative strength of the radiation zero in  $W\gamma$  production and the approximate zero in  $q_1\bar{q}_2 \rightarrow WZ$ . QCD corrections significantly affect the ratio of  $ZZ$  to  $WZ$  cross sections, but largely cancel in the  $WZ/W\gamma$  cross section ratio. Although rapidity correlations and cross section ratios are useful tools in searching for the approximate amplitude zero in  $WZ$  production, it will not be easy to establish the effect at the Tevatron or LHC, due to the limited number of  $W^\pm Z \rightarrow \ell_1^\pm \nu_1 \ell_2^+ \ell_2^-$  events expected. Our conclusions, finally, are given in Sec. V.

## II. CALCULATIONAL TOOLS

Our calculation generalizes the results of Ref. [15] to arbitrary  $C$  and  $P$  conserving  $WWZ$  couplings. It is carried out using a combination of analytic and Monte Carlo integration techniques. Details of the method can be found in Ref. [12]. The calculation is done using the narrow width approximation for the leptonically decaying  $W$  and  $Z$  bosons. In this approximation it is particularly easy to extend the NLO calculation of  $WZ$  production with on-shell  $W$  and  $Z$  bosons to include the leptonic  $W$  and  $Z$  decays. Furthermore, non-resonant Feynman diagrams such as  $d\bar{u} \rightarrow W^{-*} \rightarrow e^-\bar{\nu}_e Z$  followed by  $Z \rightarrow \mu^+\mu^-$  contribute negligibly in this limit, and thus can be ignored.

### A. Summary of $\mathcal{O}(\alpha_s)$ $WZ$ Production including $W$ and $Z$ decays

The NLO calculation of  $WZ$  production includes contributions from the square of the Born graphs, the interference between the Born graphs and the virtual one-loop diagrams, and the square of the real emission graphs. The basic idea of the method employed here is to isolate the soft and collinear singularities associated with the real emission subprocesses by partitioning phase space into soft, collinear, and finite regions. This is done by introducing theoretical soft and collinear cutoff parameters,  $\delta_s$  and  $\delta_c$ . Using dimensional regularization, the soft and collinear singularities are exposed as poles in  $\epsilon$  (the number of space-time dimensions is  $N = 4 - 2\epsilon$  with  $\epsilon$  a small number). The infrared singularities from the soft and virtual contributions are then explicitly canceled while the collinear singularities are factorized and absorbed into the definition of the parton distribution functions. The remaining contributions are finite and can be evaluated in four dimensions. The Monte Carlo program thus generates  $n$ -body (for the Born and virtual contributions) and  $(n+1)$ -body (for the real emission contributions) final state events. The  $n$ - and  $(n+1)$ -body contributions both depend on the cutoff parameters  $\delta_s$  and  $\delta_c$ , however, when these contributions are added together to form a suitably inclusive observable, all dependence on the cutoff parameters cancels. The numerical results presented in this paper are insensitive to variations of the cutoff parameters.

Except for the virtual contribution, the  $\mathcal{O}(\alpha_s)$  corrections are all proportional to the Born cross section. It is easy to incorporate the decays  $W \rightarrow \ell_1 \nu_1$  and  $Z \rightarrow \ell_2^+ \ell_2^-$  into those terms which are proportional to the Born cross section; one simply replaces  $d\hat{\sigma}^{\text{Born}}(q_1 \bar{q}_2 \rightarrow WZ)$  with  $d\hat{\sigma}^{\text{Born}}(q_1 \bar{q}_2 \rightarrow WZ \rightarrow \ell_1 \nu_1 \ell_2^+ \ell_2^-)$  in the relevant formulae. When working at the amplitude level, the  $W$  and  $Z$  decays are trivial to implement; the vector boson polarization vector  $\epsilon_\mu^V(k)$ ,  $V = W, Z$ , is simply replaced by the respective decay current  $J_\mu^V(k)$  in the amplitude. Details of the amplitude level calculations for the Born and real emission subprocesses can be found in Ref. [16]. For  $\ell_1 = \ell_2$  the amplitudes in principle should be antisymmetrized. Since the leptons originating from the decay of the  $W$  and  $Z$  bosons are usually well sepa-

rated, effects from the antisymmetrization of the amplitudes are expected to be very small and hence are ignored here.

The only term in which it is more difficult to incorporate the  $W$  and  $Z$  decays is the virtual contribution. Rather than undertake the non-trivial task of recalculating the virtual correction term for the case of leptonically decaying weak bosons, we have instead opted to use the virtual correction for real on-shell  $W$  and  $Z$  bosons which we subsequently decay ignoring spin correlations. When spin correlations are ignored, the spin summed squared matrix element factorizes into separate production and decay squared matrix elements.

Neglecting spin correlations slightly modifies the shapes of the angular distributions of the final state leptons, but does not alter the total cross section as long as no angular cuts (*e.g.*, rapidity cuts) are imposed on the final state leptons. For realistic rapidity cuts, cross sections are changed by typically 10% if spin correlations are neglected. Since the size of the finite virtual correction is less than  $\sim 10\%$  the size of the Born cross section, the overall effect of neglecting the spin correlations in the finite virtual correction is expected to be negligible compared to the combined 10%  $\sim 20\%$  uncertainty from the parton distribution functions, the choice of the scale  $Q^2$ , and higher order QCD corrections.

## B. Incorporation of Anomalous $WWZ$ Couplings

The  $WWZ$  vertex is uniquely determined in the SM by  $SU(2) \otimes U(1)$  gauge invariance. In  $WZ$  production the  $W$  and  $Z$  bosons couple to essentially massless fermions, which insures that effectively  $\partial_\mu W^\mu = 0$  and  $\partial_\mu Z^\mu = 0$ . These conditions together with Lorentz invariance and conservation of  $C$  and  $P$ , allow three free parameters,  $g_1$ ,  $\kappa$ , and  $\lambda$ , in the  $WWZ$  vertex. The most general Lorentz,  $C$ , and  $P$  invariant vertex is described by the effective Lagrangian [17]

$$\mathcal{L}_{WWZ} = -i e \cot\theta_W \left[ g_1 (W_{\mu\nu}^\dagger W^\mu Z^\nu - W_\mu^\dagger Z_\nu W^{\mu\nu}) + \kappa W_\mu^\dagger W_\nu Z^{\mu\nu} + \frac{\lambda}{M_W^2} W_{\lambda\mu}^\dagger W_\nu^\mu Z^{\nu\lambda} \right], \quad (1)$$

where  $W^\mu$  and  $Z^\mu$  are the  $W^-$  and  $Z$  fields, respectively,  $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ , and  $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ . At tree level in the SM,  $g_1 = 1$ ,  $\kappa = 1$ , and  $\lambda = 0$ . All higher dimensional operators are obtained by replacing  $V^\mu$  with  $(\partial^2)^m V^\mu$ ,  $V = W, Z$ , where  $m$  is an arbitrary positive integer, in the terms proportional to  $\Delta g_1 = g_1 - 1$ ,  $\Delta\kappa = \kappa - 1$ , and  $\lambda$ . These operators form a complete set and can be summed by replacing  $\Delta g_1$ ,  $\Delta\kappa$ , and  $\lambda$  by momentum dependent form factors. All details are contained in the specific functional form of the form factor and its scale  $\Lambda_{FF}$ . The form factor nature of  $\Delta g_1$ ,  $\Delta\kappa$ , and  $\lambda$  will be discussed in more detail later in this section.

Following the standard notation of Ref. [17], we have chosen, without loss of generality, the  $W$  boson mass,  $M_W$ , as the energy scale in the denominator of the term proportional to  $\lambda$  in Eq. (1). If a different mass scale,  $M$ , had been used, then all of our subsequent results could be obtained by scaling  $\lambda$  by a factor  $M^2/M_W^2$ .

At present, the  $WWZ$  coupling constants are only weakly constrained experimentally. The CDF Collaboration recently presented preliminary 95% confidence level (CL) limits on  $\Delta\kappa$  and  $\lambda$  from a search performed in the  $p\bar{p} \rightarrow WZ \rightarrow jj\ell^+\ell^-$  ( $\ell = e, \mu$ ) channel at large di-jet transverse momenta,  $p_T(jj) > 100$  GeV [7]:

$$-8.6 < \Delta\kappa < 9.0 \quad (\text{for } \lambda = 0), \quad -1.7 < \lambda < 1.7 \quad (\text{for } \Delta\kappa = 0). \quad (2)$$

Assuming SM  $WW\gamma$  couplings, CDF also obtained a limit on  $\Delta\kappa$  from the reactions  $p\bar{p} \rightarrow W^+W^-$ ,  $W^\pm Z \rightarrow \ell^\pm\nu jj$  with  $p_T(jj) > 130$  GeV [7,18]:

$$-1.3 < \Delta\kappa < 1.4 \quad (\text{for } \lambda = 0). \quad (3)$$

To derive these limits, a dipole form factor with scale  $\Lambda_{FF} = 1.5$  TeV was assumed (see below), however, the experimental bounds are quite insensitive to the value of  $\Lambda_{FF}$ . Although bounds on these couplings can also be extracted from low energy data and high precision measurements at the  $Z$  pole, there are ambiguities and model dependencies in the results [19,20,21]. From loop contributions to rare meson decays such as  $K_L \rightarrow \mu^+\mu^-$  [22] or  $B \rightarrow K^{(*)}\mu^+\mu^-$  [23],  $\epsilon'/\epsilon$  [24], and the  $Z \rightarrow b\bar{b}$  width [25], one estimates limits for the

non-standard  $WWZ$  couplings of  $\mathcal{O}(1 - 10)$ . No rigorous bounds can be obtained from LEP I data if correlations between different contributions to the anomalous couplings are taken fully into account. In contrast, invoking a “naturalness” argument based on chiral perturbation theory [26,27], one expects deviations from the SM of  $\mathcal{O}(10^{-2})$  or less for  $g_1$ ,  $\kappa$ , and  $\lambda$ .

If  $C$  or  $P$  violating  $WWZ$  couplings are allowed, four additional free parameters,  $g_4$ ,  $g_5$ ,  $\tilde{\kappa}$ , and  $\tilde{\lambda}$  appear in the effective Lagrangian. For simplicity, these couplings are not considered in this paper.

The Feynman rule for the  $WWZ$  vertex factor corresponding to the Lagrangian in Eq. (1) is

$$-i g_{WWZ} Q_W \Gamma_{\beta\mu\nu}(k, k_1, k_2) = -i g_{WWZ} Q_W \left[ \Gamma_{\beta\mu\nu}^{\text{SM}}(k, k_1, k_2) + \Gamma_{\beta\mu\nu}^{\text{NSM}}(k, k_1, k_2) \right], \quad (4)$$

where the labeling conventions for the four-momenta and Lorentz indices are defined by Fig. 1,  $g_{WWZ} = e \cot \theta_W$  is the  $WWZ$  coupling strength,  $Q_W$  is the electric charge of the  $W$  boson in units of the proton charge  $e$ , and the factors  $\Gamma^{\text{SM}}$  and  $\Gamma^{\text{NSM}}$  are the SM and non-standard model vertex factors:

$$\Gamma_{\beta\mu\nu}^{\text{SM}}(k, k_1, k_2) = (k_1 - k_2)_\beta g_{\nu\mu} + 2 k_\mu g_{\beta\nu} - 2 k_\nu g_{\beta\mu}, \quad (5)$$

$$\begin{aligned} \Gamma_{\beta\mu\nu}^{\text{NSM}}(k, k_1, k_2) = & \frac{1}{2} \left( \Delta g_1 + \Delta \kappa + \lambda \frac{k^2}{M_W^2} \right) (k_1 - k_2)_\beta g_{\nu\mu} \\ & - \frac{\lambda}{M_W^2} (k_1 - k_2)_\beta k_\nu k_\mu + (\Delta g_1 + \Delta \kappa + \lambda) k_\mu g_{\beta\nu} \\ & - \left( 2\Delta g_1 + \lambda \frac{M_Z^2}{M_W^2} \right) k_\nu g_{\beta\mu}. \end{aligned} \quad (6)$$

The non-standard model vertex factor is written here in terms of  $\Delta g_1 = g_1 - 1$ ,  $\Delta \kappa = \kappa - 1$ , and  $\lambda$ , which all vanish in the SM.

It is straightforward to include the non-standard model couplings in the amplitude level calculations. Using the computer algebra program FORM [28], we have computed the  $q_1 \bar{q}_2 \rightarrow WZ$  virtual correction with the modified vertex factor of Eq. (4), however, the

resulting expression is too lengthy to present here. The non-standard  $WWZ$  couplings of Eq. (1) do not destroy the renormalizability of QCD. Thus the infrared singularities from the soft and virtual contributions are explicitly canceled, and the collinear singularities are factorized and absorbed into the definition of the parton distribution functions, exactly as in the SM case.

The anomalous couplings can not be simply inserted into the vertex factor as constants because this would violate  $S$ -matrix unitarity. Tree level unitarity uniquely restricts the  $WWZ$  couplings to their SM gauge theory values at asymptotically high energies [29]. This implies that any deviation of  $\Delta g_1$ ,  $\Delta \kappa$ , or  $\lambda$  from the SM expectation has to be described by a form factor  $\Delta g_1(M_{WZ}^2, p_W^2, p_Z^2)$ ,  $\Delta \kappa(M_{WZ}^2, p_W^2, p_Z^2)$ , or  $\lambda(M_{WZ}^2, p_W^2, p_Z^2)$  which vanishes when either the square of the  $WZ$  invariant mass,  $M_{WZ}^2$ , or the square of the four-momentum of the final state  $W$  or  $Z$  ( $p_W^2$  or  $p_Z^2$ ) becomes large. In  $WZ$  production  $p_W^2 \approx M_W^2$  and  $p_Z^2 \approx M_Z^2$  even when the finite  $W$  and  $Z$  widths are taken into account. However, large values of  $M_{WZ}^2$  will be probed at future hadron colliders like the LHC and the  $M_{WZ}^2$  dependence of the anomalous couplings has to be included in order to avoid unphysical results which would violate unitarity. Consequently, the anomalous couplings are introduced via form factors [30,31]

$$\Delta g_1(M_{WZ}^2, p_W^2 = M_W^2, p_Z^2 = M_Z^2) = \frac{\Delta g_1^0}{(1 + M_{WZ}^2/\Lambda_{FF}^2)^n}, \quad (7)$$

$$\Delta \kappa(M_{WZ}^2, p_W^2 = M_W^2, p_Z^2 = M_Z^2) = \frac{\Delta \kappa^0}{(1 + M_{WZ}^2/\Lambda_{FF}^2)^n}, \quad (8)$$

$$\lambda(M_{WZ}^2, p_W^2 = M_W^2, p_Z^2 = M_Z^2) = \frac{\lambda^0}{(1 + M_{WZ}^2/\Lambda_{FF}^2)^n}, \quad (9)$$

where  $\Delta g_1^0$ ,  $\Delta \kappa^0$ , and  $\lambda^0$  are the form factor values at low energies and  $\Lambda_{FF}$  represents the scale at which new physics becomes important in the weak boson sector, *e.g.*, due to new resonances or composite structures of the  $W$  and  $Z$  bosons. In order to guarantee unitarity, it is necessary to have  $n > 1/2$  for  $\Delta \kappa$  and  $n > 1$  for  $\Delta g_1$  and  $\lambda$ . For the numerical results presented here, we use a dipole form factor ( $n = 2$ ) with a scale  $\Lambda_{FF} = 1$  TeV, unless explicitly stated otherwise. The exponent  $n = 2$  is chosen in order to suppress  $WZ$

production at energies  $\sqrt{\hat{s}} > \Lambda_{FF} \gg M_W, M_Z$ , where novel phenomena like resonance or multiple weak boson production are expected to become important.

Form factors are usually not introduced if an ansatz based on chiral perturbation theory is used. In the framework of chiral perturbation theory, the effective Lagrangian describing the anomalous vector boson self-interactions breaks down at center of mass energies above a few TeV [26,27] (typically  $4\pi v \sim 3$  TeV, where  $v \approx 246$  GeV is the Higgs field vacuum expectation value). Consequently, one has to limit the center of mass energies to values sufficiently below  $4\pi v$  in this approach.

### III. QCD CORRECTIONS AND NON-STANDARD $WWZ$ COUPLINGS

We shall now discuss the phenomenological implications of NLO QCD corrections to  $WZ$  production at the Tevatron ( $p\bar{p}$  collisions at  $\sqrt{s} = 1.8$  TeV) and the LHC ( $pp$  collisions at  $\sqrt{s} = 14$  TeV). We also consider a possible upgrade [8] of the Tevatron to  $\sqrt{s} = 3.5$  TeV (DiTevatron). We first briefly describe the input parameters, cuts, and the finite energy resolution smearing used to simulate detector response. We then discuss in detail the impact of NLO QCD corrections on the observability of non-standard  $WWZ$  couplings in  $WZ$  production at the Tevatron, DiTevatron, and LHC. To simplify the discussion, we shall concentrate on  $W^+Z$  production. In  $p\bar{p}$  collisions the rates for  $W^+Z$  and  $W^-Z$  production are equal. At  $pp$  colliders, the  $W^-Z$  cross section is about 30% smaller than that of  $W^+Z$  production. Furthermore, we shall only consider  $W^+ \rightarrow \ell_1^+ \nu_1$  and  $Z \rightarrow \ell_2^+ \ell_2^-$  decays ( $\ell_1, \ell_2 = e, \mu$ ).

#### A. Input Parameters

The numerical results presented here were obtained using the two-loop expression for  $\alpha_s$ . The QCD scale  $\Lambda_{\text{QCD}}$  is specified for four flavors of quarks by the choice of the parton distribution functions and is adjusted whenever a heavy quark threshold is crossed so that  $\alpha_s$  is a continuous function of  $Q^2$ . The heavy quark masses were taken to be  $m_b = 5$  GeV

and  $m_t = 150$  GeV, which is consistent with the bound obtained by DØ,  $m_t > 131$  GeV [32], and the value suggested by the current CDF data,  $m_t = 174 \pm 10^{+13}_{-12}$  GeV [33]. Our results are insensitive to the value chosen for  $m_t$ .

The SM parameters used in the numerical simulations are  $M_Z = 91.173$  GeV,  $M_W = 80.22$  GeV,  $\alpha(M_W) = 1/128$ , and  $\sin^2 \theta_W = 1 - (M_W/M_Z)^2$ . These values are consistent with recent measurements at LEP, SLC, the CERN  $p\bar{p}$  collider, and the Tevatron [34,35,36,37]. The soft and collinear cutoff parameters, as discussed in Sec. II A, are fixed to  $\delta_s = 10^{-2}$  and  $\delta_c = 10^{-3}$ . The parton subprocesses have been summed over  $u, d, s$ , and  $c$  quarks and the Cabibbo mixing angle has been chosen such that  $\cos^2 \theta_C = 0.95$ . The leptonic branching ratios have been taken to be  $B(W \rightarrow \ell\nu) = 0.107$  and  $B(Z \rightarrow \ell^+\ell^-) = 0.034$  and the total widths of the  $W$  and  $Z$  bosons are  $\Gamma_W = 2.12$  GeV and  $\Gamma_Z = 2.487$  GeV. Except where otherwise stated, a single scale  $Q^2 = M_{WZ}^2$ , where  $M_{WZ}$  is the invariant mass of the  $WZ$  pair, has been used for the renormalization scale  $\mu^2$  and the factorization scale  $M^2$ .

In order to get consistent NLO results it is necessary to use parton distribution functions which have been fit to next-to-leading order. In our numerical simulations we have used the Martin-Roberts-Stirling (MRS) [38] set S0' distributions with  $\Lambda_4 = 230$  MeV, which take into account the most recent NMC [39] and CCFR [40] data and are consistent with measurements of the proton structure functions at HERA [41]. For convenience, the MRS set S0' distributions have also been used for the LO calculations.

## B. Cuts

The cuts imposed in our numerical simulations are motivated by the finite acceptance of detectors. The complete set of transverse momentum ( $p_T$ ), pseudorapidity ( $\eta$ ), and separation cuts can be summarized as follows.

Tevatron	LHC
$p_T(\ell) > 20 \text{ GeV}$	$p_T(\ell) > 25 \text{ GeV}$
$\not{p}_T > 20 \text{ GeV}$	$\not{p}_T > 50 \text{ GeV}$
$ \eta(\ell)  < 2.5$	$ \eta(\ell)  < 3.0$
$\Delta R(\ell, \ell) > 0.4$	$\Delta R(\ell, \ell) > 0.4$

Here,  $\Delta R = [(\Delta\phi)^2 + (\Delta\eta)^2]^{1/2}$  is the separation in the pseudorapidity – azimuthal angle plane. The  $\Delta R(\ell, \ell)$  cut is only imposed on leptons of equal electric charge. It has only a small effect on the  $WZ$  cross section. For simplicity, identical cuts are imposed on final state electrons and muons. The large missing transverse momentum ( $\not{p}_T$ ) cut at LHC energies, which severely reduces the total  $WZ$  cross section, has been chosen to reduce potentially dangerous backgrounds from event pileup [42],  $pp \rightarrow Zb\bar{b} \rightarrow \ell_1\nu_1\ell_2^+\ell_2^- + X$ , and processes where particles outside the rapidity range covered by the detector contribute to the missing transverse momentum. Present studies [43,44] indicate that these backgrounds are under control for  $\not{p}_T > 50 \text{ GeV}$ . The total  $W^+Z$  cross section within cuts in the Born approximation at the Tevatron, DiTevatron, and LHC is 8.5 fb, 22.4 fb, and 25.9 fb, respectively. If the LHC is operated significantly below the design luminosity of  $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  [9], the background from event pileup is less severe and it may well be possible to lower the missing  $p_T$  threshold. If the  $\not{p}_T > 50 \text{ GeV}$  cut is replaced by a  $\not{p}_T > 20 \text{ GeV}$  requirement, the total LO  $W^+Z$  cross section triples to 80.8 fb.

### C. Finite Energy Resolution Effects

Uncertainties in the energy measurements of the charged leptons in the detector are simulated in our calculation by Gaussian smearing of the particle four-momentum vector with standard deviation  $\sigma$ . For distributions which require a jet definition, *e.g.*, the  $WZ + 1$  jet exclusive cross section, the jet four-momentum vector is also smeared. The standard deviation  $\sigma$  depends on the particle type and the detector. The numerical results presented

here for the Tevatron/DiTevatron and LHC center of mass energies were made using  $\sigma$  values based on the CDF [45] and ATLAS [43] specifications, respectively.

#### D. Inclusive NLO Cross Sections

The sensitivity of  $WZ$  production to anomalous  $WWZ$  couplings in the Born approximation was studied in detail in Refs. [3] and [4]. The distributions of the  $Z$  boson transverse momentum,  $p_T(Z)$ , and the  $WZ$  invariant mass,  $M_{WZ}$ , were found to be sensitive to the anomalous couplings. However, at hadron colliders the  $WZ$  invariant mass cannot be determined unambiguously because the neutrino from the  $W$  decay is not observed. If the transverse momentum of the neutrino is identified with the missing transverse momentum of a given  $WZ$  event, the unobserved longitudinal neutrino momentum  $p_L(\nu)$  can be reconstructed, albeit with a twofold ambiguity, by imposing the constraint that the neutrino and the charged lepton four-momenta combine to form the  $W$  rest mass [46,47]. Neglecting the lepton mass one finds

$$p_L(\nu) = \frac{1}{2 p_T^2(\ell)} \left\{ p_L(\ell) \left( M_W^2 + 2 \mathbf{p}_T(\ell) \cdot \not{\mathbf{p}}_T \right) \pm p(\ell) \left[ \left( M_W^2 + 2 \mathbf{p}_T(\ell) \cdot \not{\mathbf{p}}_T \right)^2 - 4 p_T^2(\ell) \not{p}_T^2 \right]^{1/2} \right\}, \quad (10)$$

where  $p_L(\ell)$  denotes the longitudinal momentum of the charged lepton. The two solutions for  $p_L(\nu)$  are used to reconstruct two values for  $M_{WZ}$ . Both values are then histogrammed, each with half the event weight.

The differential cross section for  $M_{WZ}$  in the reaction  $p\bar{p} \rightarrow W^+Z+X \rightarrow \ell_1\nu_1\ell_2^+\ell_2^-+X$  at  $\sqrt{s} = 1.8$  TeV is shown in Fig. 2. The Born and NLO results are shown in Fig. 2a and Fig. 2b, respectively. In both cases, results are displayed for the SM and for three sets of anomalous couplings, namely,  $(\lambda^0 = -0.5, \Delta\kappa^0 = \Delta g_1^0 = 0)$ ,  $(\Delta\kappa^0 = -1.0, \lambda^0 = \Delta g_1^0 = 0)$ , and  $(\Delta g_1^0 = -0.5, \Delta\kappa^0 = \lambda^0 = 0)$ . For simplicity, only one anomalous coupling at a time is allowed to differ from its SM value. The figure shows that at the Tevatron center of mass energy, NLO QCD corrections do not have a large influence on the sensitivity of the reconstructed

$WZ$  invariant mass distribution to anomalous couplings. The  $\mathcal{O}(\alpha_s)$  corrections at Tevatron energies are approximately 30% for the SM as well as for the anomalous coupling cases. Since the anomalous terms in the helicity amplitudes grow like  $\sqrt{\hat{s}}/M_W$  ( $\hat{s}/M_W^2$ ) for  $\Delta\kappa$  ( $\lambda$  and  $\Delta g_1$ ) [3], where  $\hat{s}$  denotes the parton center of mass energy squared, non-standard couplings give large enhancements in the cross section at large values of  $M_{WZ}$ .

The  $WZ$  invariant mass distributions at the DiTevatron and the LHC are shown in Figs. 3 and 4, respectively. In both cases, the sensitivity of the  $M_{WZ}$  distribution to anomalous  $WWZ$  couplings is significantly more pronounced than for  $p\bar{p}$  collisions at  $\sqrt{s} = 1.8$  TeV. NLO QCD corrections enhance the SM  $M_{WZ}$  differential cross section by about a factor 2 at the LHC, whereas the  $\mathcal{O}(\alpha_s)$  corrections at the DiTevatron are very similar in size to those found at Tevatron energies. For non-standard  $WWZ$  couplings, the QCD corrections are more modest at the LHC. Because of the form factor parameters assumed, the result for  $\Delta\kappa^0 = -1$  approaches the SM result at large values of  $M_{WZ}$ . As mentioned before, we have used  $n = 2$  and a form factor scale of  $\Lambda_{FF} = 1$  TeV in all our numerical simulations [see Eqs. (7) – (9)]. For a larger scale  $\Lambda_{FF}$ , the deviations from the SM result become more pronounced at high energies. No significant change in the shape of the  $M_{WZ}$  distribution is observed for all center of mass energies considered.

In Fig. 5 we investigate the influence of anomalous  $WWZ$  couplings on the  $WZ$  invariant mass spectrum at next-to-leading order, together with the effect of the twofold ambiguity in  $M_{WZ}$  originating from the reconstruction of the longitudinal momentum of the neutrino from the  $W$  decay, in more detail. The lower dotted, dashed, and dot-dashed lines display the  $WZ$  invariant mass distribution for  $\Delta\kappa^0 = +1$  (+1),  $\Delta g_1^0 = +0.5$  (+0.25), and  $\lambda^0 = +0.5$  (+0.25), whereas the upper curves show the  $M_{WZ}$  spectrum for  $\Delta\kappa^0 = -1$  (-1),  $\Delta g_1^0 = -0.5$  (-0.25), and  $\lambda^0 = -0.5$  (-0.25) at the Tevatron (LHC). For  $\Delta\kappa^0$  and  $\Delta g_1^0$ , negative anomalous couplings lead to significantly larger deviations from the SM prediction than positive non-standard couplings of equal magnitude, whereas there is little difference for  $\lambda^0$  (dashed lines). For  $\Delta\kappa^0$  the sign dependence is more pronounced at small energies. Other distributions, such as the cluster transverse mass distribution or the transverse momentum distribution of

the  $Z$  boson display a similar behaviour.

This effect can be easily understood from the high energy behaviour of the  $WZ$  production amplitudes,  $\mathcal{M}(\lambda_Z, \lambda_W)$ , where  $\lambda_Z$  ( $\lambda_W$ ) denotes the helicity of the  $Z$  ( $W$ ) boson [3,5]. In the SM, only  $\mathcal{M}(\pm, \mp)$  and  $\mathcal{M}(0, 0)$  remain finite for  $\hat{s} \rightarrow \infty$ . Contributions to the helicity amplitudes proportional to  $\lambda$  mostly influence the  $(\pm, \pm)$  amplitudes and increase like  $\hat{s}/M_W^2$  at large energies. The SM  $(\pm, \pm)$  amplitudes vanish like  $1/\hat{s}$ , and the non-standard terms dominate except for the threshold region,  $\sqrt{\hat{s}} \approx M_W + M_Z$ . For non-standard values of  $\lambda$ , the cross section therefore depends only very little on the sign of the anomalous coupling. Terms proportional to  $\Delta g_1$  also increase like  $\hat{s}/M_W^2$  with energy, but mostly contribute to the  $(0, 0)$  amplitude, which remains finite in the SM in the high energy limit. Interference effects between the SM and the anomalous contributions to the  $(0, 0)$  amplitude, thus, are non-negligible, resulting in a significant dependence of the cross section on the sign of  $\Delta g_1$ . Terms proportional to  $\Delta\kappa$ , finally, are proportional to  $\sqrt{\hat{s}}/M_W$  and mostly influence the amplitudes with a longitudinal  $W$  and a transverse  $Z$  boson. In the SM, these terms vanish like  $1/\sqrt{\hat{s}}$ . In the high energy limit one therefore expects little dependence of the cross section on the sign of  $\Delta\kappa$ , similar to the situation encountered for  $\lambda$  (see Fig. 5b). However, since the terms proportional to  $\Delta\kappa$  increase less drastically with energy, interference effects between those terms and the SM amplitudes are substantial near threshold.

The dash-double-dotted line in Fig. 5 shows the true  $M_{WZ}$  distribution. The distribution of the reconstructed invariant mass at both center of mass energies is harder than the true  $M_{WZ}$  distribution. At LHC energies, the twofold ambiguity in the reconstructed  $WZ$  invariant mass only slightly affects the  $M_{WZ}$  distribution (Fig. 5b). At the Tevatron, on the other hand, the true and reconstructed invariant masses are quite different for  $M_{WZ} > 500$  GeV, thus severely degrading the sensitivity to non-standard  $WWZ$  couplings. If the  $W$  decay is treated in the narrow width approximation, one of the two reconstructed invariant masses coincides with the true  $WZ$  invariant mass. Since the  $M_{WZ}$  spectrum is steeply falling, the incorrect solution of the reconstructed invariant mass influences the distribution in a noticeable way only if it is larger than the true  $WZ$  invariant mass. The average difference

(absolute value) between the two reconstructed values of  $M_{WZ}$  is almost independent of the center of mass energy. As a result, the twofold ambiguity in the reconstructed  $WZ$  invariant mass affects the  $M_{WZ}$  spectrum at the LHC much less than at the Tevatron.

As an alternative to the  $WZ$  invariant mass spectrum, the differential cross section of the cluster transverse mass  $M_T(\ell_1\ell_2^+\ell_2^-; \not{p}_T)$  [48] can be studied. The cluster transverse mass is defined by

$$M_T^2(c; \not{p}_T) = \left[ \left( M_c^2 + |\mathbf{p}_T(c)|^2 \right)^{1/2} + \not{p}_T \right]^2 - |\mathbf{p}_T(c) + \not{p}_T|^2, \quad (11)$$

where  $M_c$  denotes the invariant mass of the cluster  $c = \ell_1\ell_2^+\ell_2^-$ . The  $M_T$  distribution at the Tevatron and the LHC is shown in Fig. 6. Since QCD corrections change its shape only slightly, we only show the NLO  $M_T$  distribution. At the Tevatron, the cluster transverse mass distribution is seen to be significantly more sensitive to anomalous couplings than the reconstructed  $WZ$  invariant mass distribution, in particular for  $\lambda$  and  $\Delta g_1$ . The cluster transverse mass distribution for  $p\bar{p}$  collisions at  $\sqrt{s} = 3.5$  TeV is qualitatively very similar to that obtained at Tevatron energies and is therefore not shown.

In Figs. 7 – 9 we show the differential cross section for the transverse momentum of the  $Z$  boson,  $p_T(Z)$ . The  $p_T(Z)$  spectrum is seen to be considerably more sensitive to non-standard  $WWZ$  couplings than the cluster transverse mass or the  $WZ$  invariant mass distributions. At high transverse momentum, a large enhancement of the cross section is observed. On the other hand, at Tevatron and DiTevatron energies, the  $p_T(Z)$  differential cross section is smaller than predicted in the SM for  $p_T(Z) < 30$  GeV, if anomalous  $WWZ$  couplings are present. Due to the relatively large  $\not{p}_T$  cut imposed, the  $Z$  boson transverse momentum distribution at the LHC in the Born approximation displays a pronounced shoulder at  $p_T(Z) \approx 65$  GeV (see Fig. 9a). The  $\not{p}_T$  cut mostly affects the small  $p_T(Z)$  region ( $p_T(Z) < 100$  GeV) and therefore does not significantly reduce the sensitivity to non-standard  $WWZ$  couplings. Once NLO QCD corrections are taken into account this shoulder disappears due to contributions from the real emission subprocesses,  $q_1\bar{q}_2 \rightarrow WZg$  and  $q_1g \rightarrow WZq_2$ .

In contrast to the other distributions studied so far, the shape of the SM  $Z$  boson transverse momentum spectrum is considerably affected by NLO QCD corrections. This is demonstrated in detail in Fig. 10, where we show the ratio of the NLO and LO differential cross sections of the  $Z$  boson transverse momentum. At Tevatron and DiTevatron energies, the  $\mathcal{O}(\alpha_s)$  corrections are approximately 30% for the SM, and 35 – 40% for the anomalous coupling cases at small  $p_T(Z)$  values. In the SM case, the size of the QCD corrections increases to  $\sim 60\%$  for  $p_T(Z) = 300$  GeV at the Tevatron, and to  $\sim 80\%$  at the DiTevatron. For non-standard couplings, on the other hand, the QCD corrections are between 20% and 40% over the whole  $p_T(Z)$  range plotted. This is exemplified by the dashed and dot-dashed lines in Fig. 10a, which show the ratio of NLO to LO cross sections for  $\lambda^0 = -0.5$  at the Tevatron and DiTevatron, respectively. At the LHC (see Fig. 10b), the shape of the  $p_T(Z)$  distribution is drastically altered by the NLO QCD corrections. At  $p_T(Z) = 800$  GeV, the QCD corrections increase the SM cross section by about a factor 5, whereas the enhancement is only a factor 1.6 at  $p_T(Z) = 100$  GeV. In the presence of anomalous couplings, the higher order QCD corrections are much smaller than in the SM. In regions where the anomalous terms dominate, the  $\mathcal{O}(\alpha_s)$  corrections are typically between 20% and 40%. This is illustrated by the dashed curve in Fig. 10b, which shows the NLO to LO cross section ratio for  $\lambda^0 = -0.25$ . At next-to-leading order, the sensitivity of the  $Z$  boson transverse momentum spectrum to anomalous couplings is thus considerably reduced. The transverse momentum distribution of the  $W$  boson in the SM exhibits a similar strong sensitivity to QCD corrections. Qualitatively, the change of the shape of the  $p_T(Z)$  distribution in  $WZ$  production and of the photon transverse momentum distribution in  $W\gamma$  production at high energies [49] are very similar.

## E. Exclusive NLO QCD Corrections

The large QCD corrections at high values of  $p_T(Z)$  are caused by the combined effects of destructive interference in the Born process, a collinear enhancement factor in the  $q_1 g \rightarrow$

$WZq_2$  partonic cross section for  $p_T(Z) \gg M_W$ , and the large  $qg$  luminosity at LHC energies. In the SM, delicate cancellations between the amplitudes of the Born diagrams occur in the central rapidity region in  $WZ$  production. These cancellations are responsible for the approximate amplitude zero [5] and suppress the  $WZ$  differential cross section, in particular for large  $W$  and  $Z$  transverse momenta. In the limit  $p_T(Z) \gg M_W$ , the cross section for  $q_1g \rightarrow WZq_2$  can be obtained using the Altarelli-Parisi approximation for collinear emission. One finds [11]:

$$d\hat{\sigma}(q_1g \rightarrow WZq_2) = d\hat{\sigma}(q_1g \rightarrow q_1Z) \frac{g_W^2}{16\pi^2} \ln^2\left(\frac{p_T^2(Z)}{M_W^2}\right), \quad (12)$$

where  $g_W = e/\sin\theta_W$ . Thus, the quark gluon fusion process carries an enhancement factor  $\ln^2(p_T^2(Z)/M_W^2)$  at large values of the  $Z$  boson transverse momentum. It arises from the kinematic region where the  $Z$  boson is produced at large  $p_T$  and recoils against the quark, which radiates a soft  $W$  boson collinear to the quark. Since the Feynman diagrams entering the derivation of Eq. (12) do not involve the  $WWZ$  vertex, the logarithmic enhancement factor only affects the SM matrix elements. At the LHC, the  $p_T(Z)$  differential cross section obtained using Eq. (12) agrees within 30% with the exact  $Z$  boson transverse momentum distribution for  $p_T(Z) > 200$  GeV [11]. Together with the very large  $qg$  luminosity at supercollider energies and the suppression of the SM  $WZ$  rate at large values of the  $Z$  boson transverse momentum in the Born approximation, the logarithmic enhancement factor is responsible for the size of the inclusive NLO QCD corrections to  $WZ$  production, as well as for the change in the shape of the  $p_T(Z)$  distribution. The same enhancement factor also appears in the antiquark gluon fusion process, however, the  $\bar{q}g$  luminosity is much smaller than the  $qg$  luminosity for large values of the  $Z$  boson transverse momentum. Since the  $W$  boson does not couple directly to the gluon, the process  $q_1\bar{q}_2 \rightarrow WZg$  is not enhanced at large values of the  $Z$  boson transverse momentum. Arguments similar to those presented above also apply to the  $W$  boson transverse momentum distribution in the limit  $p_T(W) \gg M_Z$ , and a relation analogous to Eq. (12) can be derived.

From the picture outlined in the previous paragraph, one expects that, at next-to-leading

order at supercollider energies,  $WZ$  events with a high  $p_T$   $Z$  boson most of the time also contain a high transverse momentum jet. At the Tevatron, on the other hand, the fraction of high  $p_T(Z)$   $WZ$  events with a hard jet should be considerably smaller, due to the much smaller  $qg$  luminosity at lower energies. The decomposition of the inclusive SM NLO  $p_T(Z)$  differential cross section into NLO 0-jet and LO 1-jet exclusive cross sections at the Tevatron and LHC is shown in Figs. 11a and 12a, respectively. The SM NLO 0-jet  $p_T(Z)$  distribution at the two center of mass energies are compared with the corresponding distributions obtained in the Born approximation in Figs. 11b and 12b. Here, a jet is defined as a quark or gluon with

$$p_T(j) > 10 \text{ GeV} \quad \text{and} \quad |\eta(j)| < 2.5 \quad (13)$$

at the Tevatron and

$$p_T(j) > 50 \text{ GeV} \quad \text{and} \quad |\eta(j)| < 3 \quad (14)$$

at the LHC. The sum of the NLO 0-jet and the LO 1-jet exclusive cross section is equal to the inclusive NLO cross section. The results for the NLO exclusive  $WZ + 0$  jet and the LO exclusive  $WZ + 1$  jet differential cross sections depend explicitly on the jet definition. Only the inclusive NLO distributions are independent of the jet definition. It should be noted that the jet transverse momentum threshold can not be lowered to arbitrarily small values in our calculation for theoretical reasons. For transverse momenta below 5 GeV (20 GeV) at the Tevatron (LHC), soft gluon resummation effects are expected to significantly change the jet  $p_T$  distribution [50]. For the jet definitions listed above (Eqs. (13) and (14)), these effects are expected to be unimportant, and therefore are ignored in our calculation.

With the jet definition of Eq. (13), the inclusive NLO cross section at the Tevatron is composed predominately of 0-jet events (see Fig. 11a). Due to the logarithmic enhancement factor, the 1-jet cross section becomes relatively more important at large values of the  $Z$  boson transverse momentum. For  $p_T(Z)$  values above 200 GeV, the 1-jet cross section is larger than the 0-jet rate at the LHC, and dominates completely at high  $p_T(Z)$  (see Fig. 12a).

Figure 11b compares the NLO  $WZ + 0$  jet cross section with the result obtained in the Born approximation at the Tevatron. For the jet definition chosen [see Eq. (13)], the results are almost identical over the entire transverse momentum range displayed. At the LHC, the NLO  $WZ + 0$  jet result is at most 20% smaller than the cross section obtained in the Born approximation (Fig. 12b). Note that the characteristic shoulder at  $p_T(Z) \approx 65$  GeV in the Born  $p_T(Z)$  distribution, which results from the large  $\not{p}_T$  cut, is eliminated in the NLO  $WZ + 0$  jet differential cross section. The LO and NLO  $WZ + 0$  jet  $p_T(\ell_{1,2})$ ,  $\not{p}_T$ , and  $y(\ell_{1,2})$  differential cross sections also agree to better than 20% [15]. The results for DiTevatron energies are very similar to those obtained for the Tevatron.

If the jet defining  $p_T$  threshold is lowered to 30 GeV and jets can be identified out to  $|\eta(j)| = 4.5$  at the LHC, the NLO  $WZ + 0$  jet  $p_T(Z)$  differential cross section is approximately 30% smaller than the result obtained with the jet definition of Eq. (14). Present studies suggest [43,44,51] that jets fulfilling the criteria of Eq. (14) can be identified at the LHC without problems, whereas it will be difficult to reconstruct a jet with a transverse momentum of less than 30 GeV. The pseudorapidity range covered by the LHC is not expected to extend beyond  $|\eta| = 4.5$ .

The results shown so far were obtained for  $Q^2 = M_{WZ}^2$ . Since the  $WZ + 1$  jet and the  $WZ + 0$  jet cross section in the Born approximation are tree level results, they are sensitive to the choice of the factorization scale  $Q^2$ . Figure 13 displays the scale dependence of the Born, the inclusive NLO, the  $\mathcal{O}(\alpha_s)$  0-jet exclusive, and the 1-jet exclusive cross sections for the Tevatron and LHC center of mass energies. The total cross section for the reaction  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  is plotted versus the scale  $Q$ . The factorization scale  $M^2$  and the renormalization scale  $\mu^2$  have both been set equal to  $Q^2$ .

The scale dependence of the Born cross section enters only through the  $Q^2$  dependence of the parton distribution functions. The qualitative differences between the results at the Tevatron and the LHC are due to the differences between  $p\bar{p}$  versus  $pp$  scattering and the ranges of the  $x$ -values probed. At the Tevatron,  $WZ$  production in  $p\bar{p}$  collisions is dominated by valence quark interactions. The valence quark distributions decrease with  $Q^2$  for the  $x$ -

values probed at the Tevatron (typically  $x > 0.1$ ). On the other hand, at the LHC, sea quark interactions dominate in the  $pp$  process and smaller  $x$ -values are probed (typically  $x \sim 0.02$ ). The sea quark distributions increase with  $Q^2$  for the  $x$ -values probed at the LHC. Thus the Born cross section decreases with  $Q^2$  at the Tevatron but increases with  $Q^2$  at the LHC.

The scale dependence of the 1-jet exclusive cross section enters via the parton distribution functions and the running coupling  $\alpha_s(Q^2)$ . Note that the 1-jet exclusive cross section is calculated only to lowest order and thus exhibits a strong scale dependence. The dependence on  $Q$  here is dominated by the scale dependence of  $\alpha_s(Q^2)$  which is a decreasing function of  $Q^2$ . At the NLO level, the  $Q$  dependence enters not only via the parton distribution functions and the running coupling  $\alpha_s(Q^2)$ , but also through explicit factorization scale dependence in the order  $\alpha_s(Q^2)$  correction terms. The NLO 0-jet exclusive cross section is almost independent of the scale  $Q$ . It shows a non-negligible variation with the scale only in the region  $Q < 100$  GeV at the Tevatron. In the  $WZ + 0$  jet cross section, the scale dependence of the parton distribution functions is compensated by that of  $\alpha_s(Q^2)$  and the explicit factorization scale dependence in the correction terms. The  $Q$  dependence of the inclusive NLO cross section is dominated by the 1-jet exclusive component and is significantly larger than that of the NLO 0-jet cross section.

## F. Sensitivity Limits

We now study the impact that NLO QCD corrections to  $WZ$  production have on the sensitivity limits for  $\Delta g_1^0$ ,  $\Delta \kappa^0$ , and  $\lambda^0$  at the Tevatron, DiTevatron, and LHC. For the Tevatron we consider integrated luminosities of  $\int \mathcal{L} dt = 1 \text{ fb}^{-1}$ , as envisioned for the Main Injector era, and  $10 \text{ fb}^{-1}$  (TeV\*) which could be achieved through additional upgrades of the Tevatron accelerator complex [8]. In the case of the DiTevatron we assume an integrated luminosity of  $10 \text{ fb}^{-1}$ . For the LHC we consider integrated luminosities of  $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$  and  $100 \text{ fb}^{-1}$  [9]. To extract limits at the Tevatron, TeV\*, and DiTevatron, we shall sum

over both  $W$  charges. For the LHC, we only consider  $W^+Z$  production. Interference effects between  $\Delta g_1^0$ ,  $\Delta\kappa^0$ , and  $\lambda^0$  are fully incorporated in our analysis.

To derive 95% CL limits we use the  $p_T(Z)$  distribution and perform a  $\chi^2$  test [52]. In the Born approximation, the  $Z$  boson transverse momentum distribution in general yields the best sensitivity bounds. Furthermore, we use the cuts summarized in Sec. IIIB, and the jet definitions of Eqs. (13) and (14). Unless explicitly stated otherwise, a dipole form factor ( $n = 2$ ) with scale  $\Lambda_{FF} = 1$  TeV is assumed. For the Tevatron with  $1 \text{ fb}^{-1}$  we split the  $p_T(Z)$  distribution into 3 bins, whereas 7 bins are used in all other cases. In each bin the Poisson statistics are approximated by a Gaussian distribution. In order to achieve a sizable counting rate in each bin, all events with  $p_T(Z) > 60 \text{ GeV}$  ( $120 \text{ GeV}$ ) at the Tevatron (TeV\*) are collected in a single bin. Similarly, all events with  $p_T(Z) > 180 \text{ GeV}$  ( $240 \text{ GeV}$  [ $480 \text{ GeV}$ ]) at the DiTevatron (LHC with  $10 \text{ fb}^{-1}$  [ $100 \text{ fb}^{-1}$ ]) are accumulated into one bin. This procedure guarantees that a high statistical significance cannot arise from a single event at large transverse momentum, where the SM predicts, say, only 0.01 events. In order to derive realistic limits we allow for a normalization uncertainty of 50% in the SM cross section. Background contributions are expected to be small for the cuts we impose, and are ignored in our derivation of sensitivity bounds.

Our results are summarized in Figs. 14 and 15, and Tables I and II. The cross section in each bin is a bilinear function of the anomalous couplings  $\Delta\kappa^0$ ,  $\lambda^0$ , and  $\Delta g_1^0$ . Studying the correlations in the  $\Delta\kappa^0 - \lambda^0$ , the  $\Delta\kappa^0 - \Delta g_1^0$ , and the  $\Delta g_1^0 - \lambda^0$  planes is therefore sufficient to fully include all interference effects between the various  $WWZ$  couplings. Figure 14 shows 95% CL contours in the three planes for the  $p\bar{p}$  collider options obtained from the inclusive NLO  $p_T(Z)$  distribution. Table I displays the 95% CL sensitivity limits, including all correlations, at leading order and next-to-leading order for the three  $WWZ$  couplings for the process  $p\bar{p} \rightarrow W^\pm Z + X \rightarrow \ell_1^\pm \nu_1 \ell_2^+ \ell_2^- + X$ . At Tevatron and DiTevatron energies, the increase of the cross section at  $\mathcal{O}(\alpha_s)$  and the reduced sensitivity at large values of the  $Z$  boson transverse momentum balance each other, and the limits obtained at LO and NLO are usually very similar. Our limits fully reflect the strong sign dependence of the differential

cross sections observed for  $\Delta\kappa^0$  and  $\Delta g_1^0$  (see Fig. 5).

With an integrated luminosity of  $1 \text{ fb}^{-1}$  it will not be possible to perform a very precise measurement of the  $WWZ$  vertex in the  $WZ \rightarrow \ell_1\nu_1\ell_2^+\ell_2^-$  channel at the Tevatron. For integrated luminosities of less than a few  $\text{fb}^{-1}$ , the limits which can be achieved, however, may be significantly improved by combining the bounds from  $WZ \rightarrow \ell_1\nu_1\ell_2^+\ell_2^-$  with the limits obtained from  $p\bar{p} \rightarrow WZ \rightarrow \ell_1\nu_1jj$  and  $p\bar{p} \rightarrow WZ \rightarrow jj\ell_2^+\ell_2^-$  at large di-jet transverse momenta [53]. Currently, these channels are used by the CDF Collaboration to extract information on the structure of the  $WWZ$  vertex [7,18]. Decay modes where the  $W$  or  $Z$  boson decays hadronically have a considerably larger branching ratio than the  $WZ \rightarrow \ell_1\nu_1\ell_2^+\ell_2^-$  channel and thus yield higher rates. On the other hand, they are plagued by a substantial  $W/Z + 2$  jet QCD background, which, for large integrated luminosities ( $\geq 10 \text{ fb}^{-1}$ ), will eventually limit the sensitivity of the semi-hadronic  $WZ$  decay channels to anomalous  $WWZ$  couplings.

The limits which can be achieved for  $\Delta\kappa^0$  at the TeV\* from  $WZ \rightarrow \ell_1\nu_1\ell_2^+\ell_2^-$  are about a factor 1.8 better than those at the Tevatron with  $1 \text{ fb}^{-1}$ . The bounds on  $\Delta g_1^0$  and  $\lambda^0$  improve by a factor 2 to 2.7. Increasing the energy of the Tevatron to 3.5 TeV (DiTevatron) improves the limits again significantly, in particular, the bound on  $\Delta\kappa^0$ . Due to the rather strong interference effects between the SM and the anomalous terms of the helicity amplitudes for  $\Delta g_1$  and  $\Delta\kappa$ , the contours sometimes deviate substantially from the elliptical form naively expected. Furthermore, significant correlations are observed, in particular, between  $\Delta\kappa^0$  and  $\Delta g_1^0$  (see Fig. 14b). The limits obtained with a 0-jet requirement imposed are virtually identical to those resulting from the inclusive NLO  $p_T(Z)$  distribution.

The 95% CL limit contours for the LHC are shown in Fig. 15. Table II summarizes the LO and NLO sensitivity bounds which can be achieved at the LHC. At supercollider energies, the inclusive NLO QCD corrections in the SM are very large and drastically change the shape of the SM  $p_T(Z)$  distribution (see Fig. 9). As a result, NLO QCD corrections reduce the sensitivity to anomalous couplings by 20 – 40%. As the integrated luminosity increases, larger transverse momenta become accessible. The difference between the LO

and NLO sensitivity bounds for  $100 \text{ fb}^{-1}$  therefore is somewhat larger than for  $10 \text{ fb}^{-1}$ . For the parameters chosen, the inclusive NLO bounds which can be obtained from  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+ \nu_1 \ell_2^+ \ell_2^- + X$  at  $\sqrt{s} = 14 \text{ TeV}$  with  $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$  are quite similar to those which are expected from the DiTevatron for  $W^\pm Z$  production and equal integrated luminosity.

As we have seen in Sec. III E, the size of the  $\mathcal{O}(\alpha_s)$  QCD corrections at the LHC can be significantly reduced by vetoing hard jets in the central rapidity region, *i.e.*, by imposing a “zero jet” requirement and considering only the  $WZ + 0$  jet channel. A zero jet cut for example has been imposed in the CDF measurement of the ratio of  $W$  to  $Z$  cross sections [54] and the  $W$  mass measurement [55]. The sensitivity limits obtained for the  $WZ + 0$  jet channel at NLO are  $10 - 30\%$  better than those obtained in the inclusive NLO case and are quite often close to those obtained from the leading order  $p_T(Z)$  distribution (see Table II and the dotted contours in Fig. 15). The NLO  $WZ + 0$  jet differential cross section is also more stable to variations of the factorization scale  $Q^2$  than the Born and inclusive NLO  $WZ + X$  cross sections (see Fig. 13). The systematic errors which originate from the choice of  $Q^2$  will thus be smaller for bounds derived from the NLO  $WZ + 0$  jet differential cross section than those obtained from the inclusive NLO  $WZ + X$  or the Born cross section. The limits extracted from the  $WZ + 0$  jet exclusive channel depend only negligibly on the jet definition used.

The bounds which can be achieved at the LHC improve by up to a factor 3 if an integrated luminosity of  $100 \text{ fb}^{-1}$  can be achieved (dot-dashed contours in Fig. 15). Note that the  $\Delta\kappa^0$  and  $\Delta g_1^0$  limits are strongly correlated in this case. This effect is due to the relatively small form factor scale chosen ( $\Lambda_{FF} = 1 \text{ TeV}$ ), which significantly suppresses the non-standard terms in the helicity amplitudes at high energies.

At Tevatron (DiTevatron) energies, the sensitivities achievable are insensitive to the exact form and scale of the form factor for  $\Lambda_{FF} > 400 \text{ GeV}$  ( $\Lambda_{FF} > 800 \text{ GeV}$ ). At the LHC, the situation is somewhat different and the sensitivity bounds depend on the value chosen for  $\Lambda_{FF}$  [3]. This is illustrated in Table IIc, where we list the bounds which can be achieved at

the LHC with  $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$  and a form factor scale of  $\Lambda_{FF} = 3 \text{ TeV}$ . The limits for the higher scale are a factor 1.8 to 5 better than those found for  $\Lambda_{FF} = 1 \text{ TeV}$  with the same integrated luminosity. For  $\Lambda_{FF} > 3 \text{ TeV}$ , the sensitivity bounds depend only marginally on the form factor scale, due to the very rapidly falling cross section at the LHC for parton center of mass energies in the multi-TeV region. The dependence of the limits on the cutoff scale  $\Lambda_{FF}$  in the form factor can be understood easily from Fig. 9. The improvement in sensitivity with increasing  $\Lambda_{FF}$  is due to the additional events at large  $p_T(Z)$  which are suppressed by the form factor if the scale  $\Lambda_{FF}$  has a smaller value.

To a lesser degree, the bounds also depend on the power  $n$  in the form factor, which we have assumed to be  $n = 2$ . For example, the less drastic cutoff for  $n = 1$  instead of  $n = 2$  in the form factor allows for additional high  $p_T(Z)$  events and therefore leads to a slightly increased sensitivity to the low energy values  $\Delta\kappa^0$ ,  $\Delta g_1^0$ , and  $\lambda^0$ . The sensitivity bounds listed in Tables I and II can thus be taken as representative for a wide class of form factors, including the case where constant anomalous couplings are assumed for  $M_{WZ} < \Lambda_{FF}$ , but invariant masses above  $\Lambda_{FF}$  are ignored in deriving the sensitivity bounds [26].

The bounds derived in this section are quite conservative. At the LHC, the limits can easily be improved by 10 – 20% if  $W^-Z+X$  production is included. Further improvements may result from using more powerful statistical tools than the simple  $\chi^2$  test we performed [56].

#### IV. AMPLITUDE ZEROS, RAPIDITY CORRELATIONS, AND CROSS SECTION RATIOS

Recently, it has been shown that the SM amplitude for  $q_1\bar{q}_2 \rightarrow W^\pm Z$  at the Born level exhibits an approximate zero at high energies,  $\hat{s} \gg M_Z^2$ , located at [5]

$$\cos \Theta^* = \cos \Theta_0^* \approx \pm \frac{1}{3} \tan^2 \theta_W \approx \pm 0.1, \quad (15)$$

where  $\Theta^*$  is the scattering angle of the  $Z$  boson relative to the quark direction in the  $WZ$  center of mass frame. The approximate zero is the combined result of an exact zero in the

dominant helicity amplitudes  $\mathcal{M}(\pm, \mp)$ , and strong gauge cancellations in the remaining amplitudes. At high energies, only the  $(\pm, \mp)$  and  $(0, 0)$  amplitudes remain non-zero in the SM. The existence of the zero in  $\mathcal{M}(\pm, \mp)$  at  $\cos \Theta^* \approx \pm 0.1$  is a direct consequence of the contributing  $u$ - and  $t$ -channel fermion exchange diagrams and the left-handed coupling of the  $W$  boson to fermions. Unlike the  $W^\pm \gamma$  case with its massless photon kinematics, the zero has an energy dependence which is, however, rather weak for energies sufficiently above the  $WZ$  mass threshold.

In this Section, we consider possible observable consequences of the approximate zero in  $WZ$  production in hadronic collisions and the impact of NLO QCD corrections on the relevant quantities. All numerical simulations are carried out using the parameters and cuts summarized in Secs. IIIA and IIIB. For the form factor, we again assume a dipole form factor ( $n = 2$ ) with scale  $\Lambda_{FF} = 1$  TeV (see Eqs. (7) – (9)). Since the approximate amplitude zero in  $WZ$  production is similar in nature to the well-known radiation zero in  $W\gamma$  production, analogous strategies can be applied to search for observable signals. The radiation zero in  $W\gamma$  production leads to a pronounced dip in the rapidity distribution of the photon in the parton center of mass frame,  $d\sigma/dy^*(\gamma)$  [31]. The approximate zero in the  $WZ$  amplitude is therefore expected to manifest itself as a dip in the corresponding  $y^*(Z)$  distribution. Here,

$$y^*(Z) = \frac{1}{2} \ln \left( \frac{1 + \beta_Z \cos \theta^*}{1 - \beta_Z \cos \theta^*} \right), \quad (16)$$

and

$$\beta_Z = \left[ 1 - \frac{4M_Z^2 \hat{s}}{(\hat{s} - M_W^2 + M_Z^2)^2} \right]^{1/2} \quad (17)$$

where, to lowest order,  $\hat{s} = M_{WZ}^2$  is the squared parton center of mass energy, and  $\theta^*$  the scattering angle of the  $Z$  boson with respect to the beam direction in the parton center of mass rest frame. For  $pp$  collisions the dip is centered at  $y^*(Z) = 0$ . In  $p\bar{p}$  collisions, the location of the minimum is determined by  $\cos \Theta_0^*$  of Eq. (15), the average  $WZ$  invariant mass, and the fraction of events originating from sea quark collisions. As can be seen from Figs. 2 and 3, most of the cross section originates from the region  $\sqrt{\hat{s}} = 200 - 250$  GeV.

Valence quark collisions dominate at both, Tevatron and DiTevatron energies. The minimum of the  $y^*(Z)$  distribution is therefore expected to occur at  $y^*(Z) \approx \pm 0.06$ .

The  $|y^*(Z)|$  distribution at the Tevatron and the LHC in Born approximation is shown in Fig. 16. The rapidity distribution of the  $Z$  boson in the parton center of mass frame at the DiTevatron is qualitatively very similar to that found at Tevatron energies and is therefore not shown. The SM  $|y^*(Z)|$  distribution in the true  $WZ$  rest frame (dash-double-dotted curves) displays a pronounced dip at  $|y^*(Z)| = 0$ , which originates from the approximate amplitude zero. At Tevatron energies, the dip is quite significant. However, since the unobservable longitudinal neutrino momentum can only be determined with a twofold ambiguity and, on an event to event basis, one does not know which solution is the correct one, both solutions have to be considered for each event. Assigning half of the event weight to each solution, the dip in the  $|y^*(Z)|$  distribution using the reconstructed  $WZ$  rest frame is considerably filled in (solid lines). NLO QCD corrections further diminish the significance of the dip.

In Fig. 16 we also display the effect of non-standard  $WWZ$  couplings on the  $|y^*(Z)|$  distribution (using the reconstructed  $WZ$  rest frame). The lower dotted, dashed, and dot-dashed lines display the  $|y^*(Z)|$  distribution for  $\Delta\kappa^0 = +1$  (+1),  $\Delta g_1^0 = +0.5$  (+0.25), and  $\lambda^0 = +0.5$  (+0.25), whereas the upper curves show the  $|y^*(Z)|$  spectrum for  $\Delta\kappa^0 = -1$  (-1),  $\Delta g_1^0 = -0.5$  (-0.25), and  $\lambda^0 = -0.5$  (-0.25) at the Tevatron (LHC). Non-standard  $WWZ$  couplings eliminate the approximate amplitude zero [5] and, in general, tend to fill in the dip. However, due to the relatively strong interference between standard and anomalous contributions to the helicity amplitudes for  $\Delta\kappa^0$  and  $\Delta g_1^0$  at low energies, the dip may even become more pronounced for certain (positive) values of these two couplings at the Tevatron (see the lower dotted line in Fig. 16a).

From Fig. 16 it is obvious that the dip signaling the approximate amplitude zero in  $q_1\bar{q}_2 \rightarrow WZ$  will be difficult to observe in the  $|y^*(Z)|$  distribution. In  $W\gamma$  production, correlations between the rapidities of the photon and the charged lepton originating from the  $W$  decay offer better access to the SM radiation zero than the  $y^*(\gamma)$  distribution [13].

Knowledge of the neutrino longitudinal momentum,  $p_L(\nu)$ , is not required in determining these correlations, and thus event reconstruction problems originating from the two possible solutions for  $p_L(\nu)$  are automatically avoided. In  $2 \rightarrow 2$  reactions rapidity differences are invariant under boosts,  $\Delta y(\gamma, W) = y(\gamma) - y(W) = y^*(\gamma) - y^*(W)$ . One therefore expects the rapidity difference distribution,  $d\sigma/d\Delta y(\gamma, W)$ , to exhibit a dip signaling the SM radiation zero. In  $W^\pm\gamma$  production, the dominant  $W$  helicity in the SM is  $\lambda_W = \pm 1$  [57], implying that the charged lepton from  $W \rightarrow \ell\nu$  tends to be emitted in the direction of the parent  $W$ , and thus reflects most of its kinematic properties. As a result, the dip signaling the SM radiation zero manifests itself also in the  $\Delta y(\gamma, \ell) = y(\gamma) - y(\ell)$  distribution.

The corresponding  $y(Z) - y(\ell_1)$  distribution for  $W^+Z$  production in the Born approximation is shown in Fig. 17 (solid line). Analogous to the situation encountered in  $q_1\bar{q}_2 \rightarrow W\gamma$ , the approximate zero in the  $WZ$  amplitude leads to a dip in the  $y(Z) - y(W)$  distribution [11], which is located at  $y(Z) - y(W) \approx \pm 0.12$  ( $= 0$ ) for  $W^\pm Z$  production in  $p\bar{p}$  ( $pp$ ) collisions. However, in contrast to  $W\gamma$  production, none of the  $W$  helicities dominates in  $WZ$  production [57]. The charged lepton,  $\ell_1$ , originating from the  $W$  decay,  $W \rightarrow \ell_1\nu_1$ , thus only partly reflects the kinematical properties of the parent  $W$  boson. As a result, a significant part of the correlation present in the  $y(Z) - y(W)$  spectrum is lost, and only a slight dip survives in the SM  $y(Z) - y(\ell_1)$  distribution. Due to the non-zero average rapidity difference between the lepton  $\ell_1$  and the parent  $W$  boson, the location of the minimum of the  $y(Z) - y(\ell_1)$  distribution in  $p\bar{p}$  collisions is slightly shifted to  $y(Z) - y(\ell_1) \approx 0.5$ . The  $y(Z) - y(\ell_1)$  distribution at the DiTevatron is qualitatively very similar to that obtained for  $p\bar{p}$  collisions at  $\sqrt{s} = 1.8$  TeV (see Fig. 18).

The significance of the dip in the  $y(Z) - y(\ell_1)$  distribution depends to some extent on the cut imposed on  $p_T(\ell_1)$  and the missing transverse momentum. Increasing (decreasing) the cut on  $p_T(\ell_1)$  ( $\cancel{p}_T$ ) tends to increase the probability that  $\ell_1$  is emitted in the flight direction of the  $W$  boson, and thus enhances the significance of the dip. If the  $\cancel{p}_T > 50$  GeV cut at the LHC could be reduced to 20 GeV, the dip signaling the approximate zero in the  $WZ$  production amplitude would be strengthened considerably.

Although the  $Z$  boson rapidity can readily be reconstructed from the four momenta of the lepton pair  $\ell_2^+ \ell_2^-$  originating from the  $Z$  decay, it would be experimentally easier to directly study the rapidity correlations between the charged leptons originating from the  $Z \rightarrow \ell_2^+ \ell_2^-$  and  $W \rightarrow \ell_1 \nu_1$  decays. The dotted lines in Fig. 17 show the SM  $y(\ell_2^-) - y(\ell_1^+)$  distribution for  $W^+Z$  production in the Born approximation. Because none of the  $Z$  or  $W$  helicities dominates [57] in  $q_1 \bar{q}_2 \rightarrow WZ$ , the rapidities of the leptons from  $W$  and  $Z$  decays are almost completely uncorrelated, and essentially no trace of the dip signaling the approximate amplitude zero is left in the  $y(\ell_2^-) - y(\ell_1^+)$  distribution. The  $y(\ell_2^+) - y(\ell_1^+)$  spectrum almost coincides with the  $y(\ell_2^-) - y(\ell_1^+)$  distribution and is therefore not shown.

In Figs. 18 and 19 we show the influence of NLO QCD corrections and non-standard  $WWZ$  couplings (at NLO) on the  $\Delta y(Z, \ell_1) = y(Z) - y(\ell_1)$  spectrum. At Tevatron energies, the shape of the distribution is seen to be hardly influenced by the  $\mathcal{O}(\alpha_S)$  QCD corrections. At the DiTevatron, the significance of the dip is slightly reduced. At LHC energies, the dip is completely eliminated by the inclusive QCD corrections. The NLO  $WZ + 0$  jet  $\Delta y(Z, \ell_1)$  distribution, however, is very similar to the leading order rapidity difference distribution (see Fig. 19a).

The effect of anomalous  $WWZ$  couplings on the NLO  $\Delta y(Z, \ell_1)$  distribution is exemplified by the dashed and dotted lines in Fig. 18 and in Fig. 19b. Similar to the situation encountered in the  $|y^*(Z)|$  distribution, the dip in the  $\Delta y(Z, \ell_1)$  distribution at Tevatron and DiTevatron energies may be more pronounced than in the SM for certain (positive) values of  $\Delta\kappa^0$ . The shape of the  $\Delta y(Z, \ell_1)$  distribution is seen to be quite sensitive to the sign of  $\Delta\kappa^0$  (Fig. 18). The same behaviour is observed for  $\Delta g_1^0$ , whereas positive and negative values of  $\lambda^0$  lead to a very similar  $\Delta y(Z, \ell_1)$  distribution. In general, non-standard  $WWZ$  couplings tend to fill in the dip. In order not to overburden the figures, curves for  $\lambda^0$  and  $\Delta g_1^0$  are not shown in Fig. 18. If deviations from the SM prediction were to be observed, it would be difficult to determine the sign of an anomalous coupling from the shape of the  $WZ$  invariant mass distribution, the cluster transverse mass spectrum, or the  $p_T(Z)$  distribution. For  $\Delta\kappa^0$  and  $\Delta g_1^0$ , the pronounced difference in shape of the  $\Delta y(Z, \ell_1)$  distribution for positive

and negative values may aid in determining the sign. The influence of non-standard  $WWZ$  couplings on the exclusive NLO  $WZ + 0$  jet distribution is shown in Fig. 19b. Curves are only shown for positive values of the anomalous couplings.

The error bars associated with the solid curves in Figs. 18 and 19a indicate the expected statistical uncertainties for an integrated luminosity of  $10 \text{ fb}^{-1}$  at the Tevatron and DiTevatron, and for  $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$  at the LHC. It appears that the approximate zero in the  $WZ$  amplitude will be rather difficult to observe in the  $\Delta y(Z, \ell_1)$  distribution. However, if the LHC is operated below its design luminosity of  $\mathcal{L} = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ , it may be possible to reduce the  $\not{p}_T$  cut. As mentioned above, the significance of the dip in the  $\Delta y(Z, \ell_1)$  distribution increases if the missing transverse momentum cut is lowered. The smaller collision rate is at least partially compensated by the larger total cross section for the reduced  $\not{p}_T$  cut. It is thus possible that the conditions to detect the dip in the  $\Delta y(Z, \ell_1)$  distribution improve if the LHC is operated below its design luminosity. However, more detailed simulations are required before definite conclusions can be drawn.

As an alternative to rapidity correlations, cross section ratios can be studied. Many experimental uncertainties, for example those associated with the lepton detection efficiencies, or the uncertainty in the integrated luminosity are expected to cancel, at least partially, in cross section ratios. In Ref. [14], the ratio of the  $Z\gamma$  to  $W\gamma$  cross sections,  $\mathcal{R}_{Z\gamma/W\gamma}$ , was shown to reflect the radiation zero present in the SM  $W\gamma$  helicity amplitudes. Due to the radiation zero, the  $W\gamma$  cross section is reduced in the central rapidity region. With increasing photon transverse momenta, events become more and more central in rapidity. The reduction of the  $W\gamma$  cross section at small rapidities originating from the radiation zero thus becomes more pronounced at high  $p_T(\gamma)$ . This causes the photon transverse momentum distribution of  $q_1\bar{q}_2 \rightarrow W\gamma$  to fall significantly faster than the  $p_T(\gamma)$  spectrum of  $q\bar{q} \rightarrow Z\gamma$  where no radiation zero is present. As a result,  $\mathcal{R}_{Z\gamma/W\gamma}$  increases rapidly with the minimum transverse momentum of the photon.

In Fig. 20 we study the cross section ratio

$$\mathcal{R}_{ZZ/WZ} = \frac{B^2(Z \rightarrow \ell^+ \ell^-) \sigma(ZZ)}{B(Z \rightarrow \ell^+ \ell^-) B(W \rightarrow \ell \nu) \sigma(W^\pm Z)} = \frac{B(Z \rightarrow \ell^+ \ell^-) \sigma(ZZ)}{B(W \rightarrow \ell \nu) \sigma(W^\pm Z)}, \quad (18)$$

as a function of the minimum transverse momentum of the  $Z$  boson,  $p_T^{\min}$ . To calculate the  $ZZ$  cross section, we use the results of Ref. [15] and assume the SM to be valid. The  $ZZ$  helicity amplitudes do not exhibit any zeros, whereas the SM  $WZ$  amplitude shows an approximate zero in the central rapidity region. The situation is thus qualitatively very similar to that encountered in the ratio of  $Z\gamma$  to  $W\gamma$  cross sections, and one expects  $\mathcal{R}_{ZZ/WZ}$  to grow with  $p_T^{\min}$ . Figure 20a demonstrates that, at Tevatron energies,  $\mathcal{R}_{ZZ/WZ}$  indeed rises quickly for  $p_T^{\min} > 100$  GeV in the SM, indicating the presence of the approximate zero in the  $WZ$  amplitude. For smaller values of the minimum  $Z$  boson transverse momentum,  $\mathcal{R}_{ZZ/WZ}$  is approximately constant. In the low  $p_T^{\min}$  region, the shape of the  $p_T(Z)$  distribution is dominated by  $Z$  mass effects which are similar in both processes. The cross section ratio at next-to-leading order differs only by about 10% from the LO ratio.

At LHC energies, the situation is more complex. For  $p_T^{\min} < 100$  GeV,  $\mathcal{R}_{ZZ/WZ}$  drops sharply due to the large  $p_T$  cut imposed, which significantly suppresses the  $WZ$  cross section. While the cross section ratio slowly rises with  $p_T^{\min}$  for  $p_T^{\min} > 100$  GeV at leading order,  $\mathcal{R}_{ZZ/WZ}$  continues to decrease if inclusive NLO QCD corrections are taken into account (Fig. 20b). The relatively slower rise of  $\mathcal{R}_{ZZ/WZ}$  at LO at the LHC is due to the larger fraction of the cross sections originating from sea quark collisions, and the different  $x$ -ranges probed at the Tevatron and LHC. For  $p_T^{\min}(Z) = 1$  TeV, the inclusive NLO cross section ratio is about a factor 3 smaller than  $\mathcal{R}_{ZZ/WZ}$  at leading order. At large values of the  $Z$  boson transverse momentum, the QCD corrections to  $WZ$  production at LHC energies are substantially larger than in the  $ZZ$  case [15], resulting in a large discrepancy between the LO and NLO prediction for  $\mathcal{R}_{ZZ/WZ}$ . In contrast to the situation encountered at the Tevatron, higher order QCD corrections completely blur the signal of the approximate amplitude zero in the  $WZ$  channel. Their size, however, can be substantially reduced by imposing a zero jet requirement (see Sec. IIIE and Ref. [15]). The result for the  $ZZ + 0$  jet to  $W^\pm Z + 0$  jet cross section ratio at the LHC is given by the dotted line in Fig. 20b. With a jet veto imposed, the

NLO  $ZZ$  to  $WZ$  cross section ratio rises with the minimum  $Z$  boson transverse momentum for  $p_T^{\min} > 100$  GeV, and differs by at most 15% from the LO prediction. At the Tevatron, the NLO 0-jet cross section ratio virtually coincides with the ratio obtained at LO.

The dot-dashed curve in Fig. 20, finally, shows the  $ZZ$  to  $WZ$  cross section ratio for  $\Delta\kappa^0 = +1$ , illustrating the behaviour of  $\mathcal{R}_{ZZ/WZ}$  in presence of anomalous  $WWZ$  couplings. At the Tevatron (Fig. 20a), the dot-dashed curve has been calculated taking into account inclusive  $\mathcal{O}(\alpha_s)$  QCD corrections. At LHC energies (Fig. 20b) the NLO  $ZZ$  to  $WZ$  cross section ratio is plotted with a jet veto included. Non-standard couplings lead to an enhancement of the  $WZ$  cross section, in particular at large values of  $p_T(Z)$  and, at the Tevatron,  $\mathcal{R}_{ZZ/WZ}$  decreases with  $p_T^{\min}$ . Due to the form factor parameters assumed ( $n = 2$  and  $\Lambda_{FF} = 1$  TeV), the cross section ratio at the LHC displays a broad minimum at  $p_T^{\min}(Z) \approx 300$  GeV, and increases quickly at large values of  $p_T^{\min}$ . For larger values of  $\Lambda_{FF}$ , and/or non-zero values of  $\Delta g_1^0$  or  $\lambda^0$ ,  $\mathcal{R}_{ZZ/WZ}$  rises more slowly, or may even decrease with  $p_T^{\min}(Z)$ . In general, the  $ZZ$  to  $WZ$  cross section ratio as a function of the minimum  $Z$  boson transverse momentum differs substantially in shape from the SM prediction for  $\mathcal{R}_{ZZ/WZ}$  in presence of non-standard  $WWZ$  couplings.

At the Tevatron, the limited number of  $ZZ$  and  $WZ$  events expected in the purely leptonic channels will unfortunately limit the usefulness of  $\mathcal{R}_{ZZ/WZ}$ . Even for an integrated luminosity of  $10 \text{ fb}^{-1}$  only a handful of events are expected for  $p_T(Z) > 150$  GeV, and it will be very difficult to establish the growth with  $p_T^{\min}(Z)$  predicted by the SM. At the LHC, the statistical errors are expected to be much smaller, however, one can only hope to observe the rise of  $\mathcal{R}_{ZZ/WZ}$  signalling the presence of the approximate zero in the  $WZ$  channel if a 0-jet requirement is imposed. Moreover, the rise of the cross section ratio is very slow, and for  $p_T^{\min} = 600$  GeV only about 5 (2) purely leptonic  $W^\pm Z$  ( $ZZ$ ) events are expected for  $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$ . Combined, these effects will make it quite difficult to accurately determine the slope of  $\mathcal{R}_{ZZ/WZ}$ .

A cross section ratio which suffers somewhat less from the small number of events expected at the Tevatron, and which is less sensitive to QCD corrections at the LHC is the

ratio of  $WZ$  to  $W\gamma$  cross sections,

$$\mathcal{R}_{WZ/W\gamma} = \frac{B(Z \rightarrow \ell^+ \ell^-) B(W \rightarrow \ell \nu) \sigma(W^\pm Z)}{B(W \rightarrow \ell \nu) \sigma(W^\pm \gamma)} = B(Z \rightarrow \ell^+ \ell^-) \frac{\sigma(W^\pm Z)}{\sigma(W^\pm \gamma)} \quad (19)$$

considered as a function of the minimum transverse momentum of the  $Z$  boson and photon,  $p_T^{\min}$ , respectively.  $\mathcal{R}_{WZ/W\gamma}$  measures the relative strength of the approximate zero in  $q_1 \bar{q}_2 \rightarrow WZ$  and the radiation zero in  $W\gamma$  production. Figure 21 shows the ratio  $\mathcal{R}_{WZ/W\gamma}$  as a function of  $p_T^{\min}$  for the Tevatron (part a) and LHC (part b) center of mass energies. In obtaining  $\mathcal{R}_{WZ/W\gamma}$ , we have considered both the electron and muon decay channels of the  $W$  and  $Z$  bosons. The  $W\gamma$  cross section in Fig. 21 has been calculated using the results of Ref. [49] and the following cuts on the photon:

$$\begin{aligned} p_T(\gamma) &> 10 \text{ GeV}, & |\eta(\gamma)| &< 1, \quad (\text{Tevatron}) \\ p_T(\gamma) &> 100 \text{ GeV}, & |\eta(\gamma)| &< 2.5, \quad (\text{LHC}) \\ M_T(\ell\gamma; \not{p}_T) &> 90 \text{ GeV}, & \Delta R(\gamma, \ell) &> 0.7. \end{aligned} \quad (20)$$

At small values of  $p_T^{\min}$ , the  $WZ$  to  $W\gamma$  cross section ratio rises very rapidly, due to the finite  $Z$  mass effects which dominate the shape of the  $p_T(Z)$  spectrum in this region for  $WZ$  production. For  $p_T^{\min} > 100$  GeV (200 GeV) at the Tevatron (LHC),  $\mathcal{R}_{WZ/W\gamma}$  is almost constant and independent of the center of mass energy, indicating that the radiation zero in  $q_1 \bar{q}_2 \rightarrow W\gamma$  and the approximate amplitude zero in  $WZ$  production affect the respective photon and  $Z$  boson transverse momentum distribution in a very similar way. At the Tevatron, NLO QCD corrections reduce  $\mathcal{R}_{WZ/W\gamma}$  by about 10%. At LHC energies, the individual  $\mathcal{O}(\alpha_s)$  QCD corrections are very large for both  $WZ$  and  $W\gamma$  production [49], in particular at high transverse momenta (see Figs. 9 and 12). In the cross section ratio, these large corrections cancel almost completely. For  $p_T^{\min} > 200$  GeV, QCD corrections reduce  $\mathcal{R}_{WZ/W\gamma}$  by 20% or less. In contrast to the LO cross section ratio, which is completely flat for  $p_T^{\min} > 200$  GeV,  $\mathcal{R}_{WZ/W\gamma}$  at NLO slowly rises with  $p_T^{\min}$  at the LHC.

Most theoretical models with non-standard  $WWZ$  couplings also predict anomalous  $WW\gamma$  couplings at the same time (see *e.g.*, Ref. [21]). The effects of anomalous  $WWZ$

and  $WW\gamma$  couplings may cancel almost completely in  $\mathcal{R}_{WZ/W\gamma}$  if the  $WWZ$  and  $WW\gamma$  couplings are similar in magnitude and originate from operators of the same dimension. This is illustrated by the dot-dashed and dotted lines in Fig. 21, which show  $\mathcal{R}_{WZ/W\gamma}$  at LO and NLO for  $\Delta\kappa_\gamma^0 = \Delta\kappa^0 = -1$ . Here the anomalous  $WW\gamma$  coupling  $\Delta\kappa_\gamma$  is defined through an effective Lagrangian analogous to that of Eq. (1), and we assume equal form factor scales and powers ( $\Lambda_{FF} = 1$  TeV and  $n = 2$ ) for  $\Delta\kappa$  and  $\Delta\kappa_\gamma$ . Both couplings correspond to operators of dimension four in the effective Lagrangian. Although the individual  $p_T(\gamma)$  and  $p_T(Z)$  differential cross sections are enhanced by up to one order of magnitude (see *e.g.*, Figs. 7 and 9),  $\mathcal{R}_{WZ/W\gamma}$  agrees to better than 20% with the NLO SM cross section ratio for  $\Delta\kappa_\gamma^0 = \Delta\kappa^0 = -1$ .

At DiTevatron energies, the results for  $\mathcal{R}_{ZZ/WZ}$  and  $\mathcal{R}_{WZ/W\gamma}$  are qualitatively similar to those obtained for Tevatron and are therefore not shown.

## V. SUMMARY

$WZ$  production in hadronic collisions provides an opportunity to probe the structure of the  $WWZ$  vertex in a direct and essentially model independent way. Previous studies of  $p\bar{p}^\rightarrow \rightarrow W^\pm Z$  [3,4] have been based on leading order calculations. In this paper we have presented an  $\mathcal{O}(\alpha_s)$  calculation of the reaction  $p\bar{p}^\rightarrow \rightarrow W^\pm Z + X \rightarrow \ell_1^\pm \nu_1 \ell_2^\pm \ell_2^- + X$  for general,  $C$  and  $P$  conserving,  $WWZ$  couplings, using a combination of analytic and Monte Carlo integration techniques. The leptonic decays  $W \rightarrow \ell_1 \nu_1$  and  $Z \rightarrow \ell_2^+ \ell_2^-$  have been included in the narrow width approximation in our calculation. Decay spin correlations are correctly taken into account in our approach, except in the finite virtual contribution. The finite virtual correction term contributes only at the few per cent level to the total NLO cross section, thus decay spin correlations can be safely ignored here.

The  $p_T(Z)$  differential cross section is very sensitive to non-standard  $WWZ$  couplings. We found that QCD corrections significantly change the shape of this distribution at very high energies (see Fig. 9 and 10b). This shape change is due to a combination of destructive

interference in the  $WZ$  Born subprocess and a logarithmic enhancement factor in the  $qg$  and  $\bar{q}g$  real emission subprocesses. The destructive interference suppresses the size of the  $WZ$  Born cross section and is also responsible for the approximate amplitude zero in  $q_1\bar{q}_2 \rightarrow WZ$  [5]. The logarithmic enhancement factor originates in the high  $p_T(Z)$  [ $p_T(W)$ ] region of phase space where the  $Z$  [ $W$ ] boson is balanced by a high  $p_T$  quark which radiates a soft  $W$  [ $Z$ ] boson. The logarithmic enhancement factor and the large gluon density at high center of mass energies make the  $\mathcal{O}(\alpha_s)$  corrections large for  $p_T(Z) \gg M_Z$ . Since the Feynman diagrams responsible for the enhancement at large  $p_T(Z)$  do not involve the  $WWZ$  vertex, inclusive NLO QCD corrections to  $W^\pm Z$  production tend to reduce the sensitivity to non-standard couplings. QCD corrections in  $WZ$  production thus exhibit the same features which characterize the  $\mathcal{O}(\alpha_s)$  corrections in  $W\gamma$  production.

At the Tevatron and DiTevatron,  $WZ$  production proceeds mainly via quark-antiquark annihilation and, for the expected integrated luminosities ( $\leq 10 \text{ fb}^{-1}$ ), large transverse momenta are not accessible. As a result, the sensitivity reduction in the high  $p_T$  tail caused by the QCD corrections is balanced by the larger cross section at  $\mathcal{O}(\alpha_s)$ , and the limits derived from the NLO and LO  $p_T(Z)$  distribution are very similar (see Table I). At the LHC, however, where the  $qg$  luminosity is very high and the change in slope of the SM  $p_T(Z)$  distribution from QCD corrections is very pronounced, the sensitivity bounds which can be achieved are weakened by up to 40% (see Table II).

The size of the QCD corrections at large  $p_T(Z)$  may be reduced substantially, and a fraction of the sensitivity to anomalous  $WWZ$  couplings which was lost at the LHC may be regained, by imposing a jet veto, *i.e.*, by considering the exclusive  $WZ + 0$  jet channel instead of inclusive  $WZ + X$  production. The improvement is equivalent to roughly a factor 1.5 – 2.5 increase in integrated luminosity. The dependence of the NLO  $WZ + 0$  jet cross section on the factorization scale  $Q^2$  is significantly reduced compared to that of the inclusive NLO  $WZ + X$  cross section. Uncertainties which originate from the variation of  $Q^2$  will thus be smaller for sensitivity bounds obtained from the  $WZ + 0$  jet channel than for those derived from the inclusive NLO  $WZ + X$  cross section. At the Tevatron (DiTevatron), NLO

QCD corrections do not influence the sensitivity limits in a significant way. Nevertheless, it will be important to take these corrections into account when extracting information on the structure of the  $WWZ$  vertex, in order to reduce systematic and theoretical errors.

At the Tevatron (DiTevatron) with  $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$ , taking into account all correlations between the different  $WWZ$  couplings,  $\Delta\kappa^0$  can be measured with 70 – 100% (50 – 60%) accuracy in  $WZ$  production in the purely leptonic channels, whereas the two other couplings can be determined with an uncertainty of 0.1 – 0.25. At the LHC with  $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$ ,  $\Delta\kappa^0$  can be determined with an uncertainty of about 10%, whereas  $\Delta g_1^0$  and  $\lambda^0$  can be measured to better than 0.01, with details depending on the form factor scale assumed (see Table II).

The bounds listed in Tables I and II should be compared with the limits which can be obtained in other channels, and at  $e^+e^-$  colliders. Assuming SM  $WW\gamma$  couplings,  $\Delta g_1^0$  and  $\lambda^0$  can be measured in  $p\bar{p} \rightarrow W^+W^-$ ,  $W^\pm Z \rightarrow \ell^\pm \nu jj$  with a precision similar to that which can be achieved in the  $W^\pm Z \rightarrow \ell_1^\pm \nu_1 \ell_2^\pm \ell_2^-$  mode, both at the Tevatron with  $1 \text{ fb}^{-1}$  and the TeV\*. The limits which can be obtained for  $\Delta\kappa^0$  from the  $\ell^\pm \nu jj$  final state are about a factor 4 better than those from double leptonic  $WZ$  decays [53]. The bounds which can be achieved for  $\Delta\kappa^0$ ,  $\Delta g_1^0$  and  $\lambda^0$  in  $e^+e^- \rightarrow W^+W^-$  at LEP II depend quite sensitively on the center of mass energy. For  $\sqrt{s} = 176 \text{ GeV}$  and  $\int \mathcal{L} dt = 500 \text{ pb}^{-1}$ , the  $WWZ$  couplings can be measured with a precision of about  $\pm 0.5$ , if correlations between the three couplings are taken into account [53,58]. At a linear  $e^+e^-$  collider with a center of mass energy of 500 GeV or higher, they can be determined with an accuracy of better than 0.01 [53,59].

We also studied possible experimental signals of the approximate zero in the SM  $WZ$  amplitude. Unlike the situation encountered in  $W\gamma$  production where the radiation zero leads to a pronounced minimum in the photon-lepton rapidity difference distribution, the approximate amplitude zero in  $WZ$  production causes a slight dip only in the corresponding  $\Delta y(Z, \ell_1) = y(Z) - y(\ell_1)$  distribution. In  $W^\pm\gamma$  production, the dominant  $W$  helicity is  $\lambda_W = \pm 1$ , implying that the charged lepton from the decaying  $W$  boson tends to be emitted in the direction of the parent  $W$  boson, and thus reflects most of its kinematic properties. In contrast, none of the  $W$  helicities dominates in  $WZ$  production. The charged lepton

originating from the  $W$  boson decay,  $W \rightarrow \ell_1 \nu_1$ , thus only partly reflects the kinematic properties of the parent  $W$  boson, which reduces the significance of the dip. At Tevatron and DiTevatron energies, higher order QCD corrections only negligibly influence the shape of the  $\Delta y(Z, \ell_1)$  distribution. At the LHC, however, NLO QCD effects completely obscure the dip, unless a 0-jet requirement is imposed.

Alternatively, cross section ratios can be used to search for experimental footprints of the approximate amplitude zero. We found that the ratio of  $ZZ$  to  $WZ$  cross sections as a function of the minimum  $Z$  boson transverse momentum,  $p_T^{\min}$ , increases with  $p_T^{\min}$  for values larger than 100 GeV. The increase of the ratio of  $ZZ$  to  $WZ$  cross sections is a direct consequence of the approximate zero. The ratio of  $WZ$  to  $W\gamma$  cross sections, on the other hand, is almost independent of the minimum  $p_T$  of the  $Z$  boson and photon for sufficiently large values of  $p_T^{\min}$ , indicating that the approximate zero in  $WZ$  production and the radiation zero in  $W\gamma$  production affect the  $Z$  boson and photon transverse momentum distributions in a very similar way. QCD corrections have a significant impact on the  $ZZ$  to  $WZ$  cross section ratio at the LHC unless a jet veto is imposed, whereas they largely cancel in the  $WZ$  to  $W\gamma$  cross section ratio.

Together with the  $\Delta y(Z, \ell_1)$  distribution, the  $ZZ$  to  $W^\pm Z$  and  $WZ$  to  $W\gamma$  cross section ratios are useful tools in searching for the approximate amplitude zero in  $WZ$  production. However, for the integrated luminosities envisioned, it will not be easy to conclusively establish the approximate amplitude zero in  $WZ$  production at the Tevatron or the LHC.

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## TABLES

TABLE I. Sensitivities achievable at the 95% confidence level (CL) for the anomalous  $WWZ$  couplings  $\Delta g_1^0$ ,  $\Delta\kappa^0$ , and  $\lambda^0$  in  $p\bar{p} \rightarrow W^\pm Z + X \rightarrow \ell_1^\pm \nu_1 \ell_2^\pm \ell_2^- + X$  at a) leading order and b) next-to-leading order for the Tevatron, the TeV\* ( $\sqrt{s} = 1.8$  TeV in both cases), and the DiTevatron ( $\sqrt{s} = 3.5$  TeV). The limits for each coupling apply for arbitrary values of the two other couplings. For the form factors we use Eqs. (7), (8), and (9) with  $n = 2$  and  $\Lambda_{FF} = 1$  TeV. The cuts summarized in Sec. IIIB are imposed.

a) leading order			
	Tevatron $\int \mathcal{L} dt = 1 \text{ fb}^{-1}$	TeV* $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$	DiTevatron $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$
$\Delta g_1^0$	+0.52	+0.26	+0.15
	-0.29	-0.10	-0.05
$\Delta\kappa^0$	+1.9	+1.0	+0.5
	-1.4	-0.7	-0.5
$\lambda^0$	+0.34	+0.15	+0.08
	-0.37	-0.15	-0.07

  

b) next-to-leading order			
	Tevatron $\int \mathcal{L} dt = 1 \text{ fb}^{-1}$	TeV* $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$	DiTevatron $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$
$\Delta g_1^0$	+0.50	+0.25	+0.15
	-0.27	-0.10	-0.05
$\Delta\kappa^0$	+1.8	+1.0	+0.6
	-1.2	-0.7	-0.5
$\lambda^0$	+0.33	+0.15	+0.08
	-0.34	-0.14	-0.08

TABLE II. Sensitivities achievable at the 95% confidence level (CL) for the anomalous  $WWZ$  couplings  $\Delta g_1^0$ ,  $\Delta\kappa^0$ , and  $\lambda^0$  in  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at the LHC ( $\sqrt{s} = 14$  TeV). The limits for each coupling apply for arbitrary values of the two other couplings. For the form factors we use Eqs. (7), (8), and (9) with  $n = 2$ . The cuts summarized in Sec. IIIB are imposed. In the NLO 0-jet case we have used the jet definition of Eq. (14).

a) $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$ , $\Lambda_{FF} = 1 \text{ TeV}$			
coupling	Born appr.	incl. NLO	NLO 0-jet
$\Delta g_1^0$	+0.108	+0.127	+0.115
	-0.036	-0.044	-0.038
$\Delta\kappa^0$	+0.53	+0.62	+0.60
	-0.46	-0.59	-0.49
$\lambda^0$	+0.058	+0.065	+0.063
	-0.057	-0.073	-0.062

  

b) $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$ , $\Lambda_{FF} = 1 \text{ TeV}$			
coupling	Born appr.	incl. NLO	NLO 0-jet
$\Delta g_1^0$	+0.082	+0.096	+0.085
	-0.014	-0.018	-0.016
$\Delta\kappa^0$	+0.17	+0.24	+0.20
	-0.34	-0.41	-0.35
$\lambda^0$	+0.038	+0.042	+0.036
	-0.036	-0.048	-0.038

  

c) $\int \mathcal{L} dt = 100 \text{ fb}^{-1}$ , $\Lambda_{FF} = 3 \text{ TeV}$			
coupling	Born appr.	incl. NLO	NLO 0-jet
$\Delta g_1^0$	+0.0164	+0.0200	+0.0188
	-0.0048	-0.0066	-0.0050
$\Delta\kappa^0$	+0.092	+0.128	+0.108
	-0.120	-0.160	-0.132
$\lambda^0$	+0.0084	+0.0102	+0.0092
	-0.0082	-0.0100	-0.0090

## FIGURES

FIG. 1. Feynman rule for the general  $WWZ$  vertex. The factor  $g_{WWZ} = e \cot \theta_W$  is the  $WWZ$  coupling strength and  $Q_W$  is the electric charge of the  $W$  boson. The vertex function  $\Gamma_{\beta\mu\nu}(k, k_1, k_2)$  is given in Eq. (4).

FIG. 2. The inclusive differential cross section for the reconstructed  $WZ$  mass in the reaction  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 1.8$  TeV; a) in the Born approximation and b) including NLO QCD corrections. The curves are for the SM (solid lines),  $\lambda^0 = -0.5$  (dashed lines),  $\Delta\kappa^0 = -1.0$  (dotted lines), and  $\Delta g_1^0 = -0.5$  (dot-dashed lines). The cuts imposed are summarized in Sec. IIIB.

FIG. 3. The inclusive differential cross section for the reconstructed  $WZ$  mass in the reaction  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 3.5$  TeV; a) in the Born approximation and b) including NLO QCD corrections. The curves are for the SM (solid lines),  $\lambda^0 = -0.5$  (dashed lines),  $\Delta\kappa^0 = -1.0$  (dotted lines), and  $\Delta g_1^0 = -0.5$  (dot-dashed lines). The cuts imposed are summarized in Sec. IIIB.

FIG. 4. The inclusive differential cross section for the reconstructed  $WZ$  mass in the reaction  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV; a) in the Born approximation and b) including NLO QCD corrections. The curves are for the SM (solid lines),  $\lambda^0 = -0.25$  (dashed lines),  $\Delta\kappa^0 = -1.0$  (dotted lines), and  $\Delta g_1^0 = -0.25$  (dot-dashed lines). The cuts imposed are summarized in Sec. IIIB.

FIG. 5. The inclusive NLO differential cross section for the reconstructed  $WZ$  mass in the reaction a)  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 1.8$  TeV and b)  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV. The curves are for the SM with reconstructed invariant mass (solid lines) and non-standard  $WWZ$  couplings (dotted, dashed, and dot-dashed curves) as listed on the figure. The lower (upper) lines apply for positive (negative) anomalous couplings. The dash-double-dotted line shows the true SM  $WZ$  invariant mass distribution. The cuts imposed are summarized in Sec. IIIB.

FIG. 6. The inclusive NLO differential cross section for the cluster transverse mass for a)  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 1.8$  TeV and b)  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV. The curves are for the SM (solid lines) and non-standard  $WWZ$  couplings (dotted, dashed, and dot-dashed curves) as listed on the figure. The cuts imposed are summarized in Sec. IIIB.

FIG. 7. The inclusive differential cross section for the transverse momentum of the  $Z$  boson in the reaction  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 1.8$  TeV; a) in the Born approximation and b) including NLO QCD corrections. The curves are for the SM (solid lines),  $\lambda^0 = -0.5$  (dashed lines),  $\Delta\kappa^0 = -1.0$  (dotted lines), and  $\Delta g_1^0 = -0.5$  (dot-dashed lines). The cuts imposed are summarized in Sec. IIIB.

FIG. 8. The inclusive differential cross section for the transverse momentum of the  $Z$  boson in the reaction  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 3.5$  TeV; a) in the Born approximation and b) including NLO QCD corrections. The curves are for the SM (solid lines),  $\lambda^0 = -0.5$  (dashed lines),  $\Delta\kappa^0 = -1.0$  (dotted lines), and  $\Delta g_1^0 = -0.5$  (dot-dashed lines). The cuts imposed are summarized in Sec. IIIB.

FIG. 9. The inclusive differential cross section for the transverse momentum of the  $Z$  boson in the reaction  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV; a) in the Born approximation and b) including NLO QCD corrections. The curves are for the SM (solid lines),  $\lambda^0 = -0.25$  (dashed lines),  $\Delta\kappa^0 = -1.0$  (dotted lines), and  $\Delta g_1^0 = -0.25$  (dot-dashed lines). The cuts imposed are summarized in Sec. IIIB.

FIG. 10. Ratio of the NLO to LO differential cross sections of the  $Z$  boson transverse momentum as a function of  $p_T(Z)$  for a)  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  and b)  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV. In part a) the solid (dotted) and dashed (dot-dashed) lines give the ratio in the SM and for  $\lambda^0 = -0.5$  at the Tevatron (DiTevatron), respectively. In part b) the solid and dashed curves show the ratio for the SM and  $\lambda^0 = -0.25$  at the LHC. The cuts imposed are summarized in Sec. IIIB.

FIG. 11. The differential cross section for the  $Z$  boson transverse momentum in the reaction  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 1.8$  TeV in the SM. a) The inclusive NLO differential cross section (solid line) is shown, together with the  $\mathcal{O}(\alpha_s)$  0-jet (dotted line) and the (LO) 1-jet (dot dashed line) exclusive differential cross sections, using the jet definition in Eq. (13). b) The NLO  $WZ + 0$  jet exclusive differential cross section (dotted line) is compared with the Born differential cross section (dashed line). The cuts imposed are summarized in Sec. IIIB.

FIG. 12. The differential cross section for the  $Z$  boson transverse momentum in the reaction  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV in the SM. a) The inclusive NLO differential cross section (solid line) is shown, together with the  $\mathcal{O}(\alpha_s)$  0-jet (dotted line) and the (LO) 1-jet (dot dashed line) exclusive differential cross sections, using the jet definition in Eq. (14). b) The NLO  $WZ + 0$  jet exclusive differential cross section (dotted line) is compared with the Born differential cross section (dashed line). The cuts imposed are summarized in Sec. IIIB.

FIG. 13. The total cross section for  $p\bar{p}^{(\ominus)} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  in the SM versus the scale  $Q$ ; a) at the Tevatron and b) at the LHC. The curves represent the inclusive NLO (solid lines), the Born (dot-dashed lines), the LO 1-jet exclusive (dashed lines), and the NLO 0-jet exclusive (dotted lines) cross sections. The cuts imposed are summarized in Sec. IIIB. For the jet definitions, we have used Eqs. (13) and (14).

FIG. 14. Limit contours at the 95% CL for  $p\bar{p} \rightarrow W^\pm Z + X \rightarrow \ell_1^\pm\nu_1\ell_2^+\ell_2^- + X$  derived from the inclusive NLO  $p_T(Z)$  distribution. Contours are shown in three planes: a) the  $\Delta\kappa^0 - \lambda^0$  plane, b) the  $\Delta\kappa^0 - \Delta g_1^0$  plane, and c) the  $\Delta g_1^0 - \lambda^0$  plane. The solid and dashed lines give the results for the Tevatron ( $\sqrt{s} = 1.8$  TeV) with  $\int \mathcal{L} dt = 1 \text{ fb}^{-1}$  and  $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$ , respectively. The dotted curve shows the result obtained for the DiTevatron ( $\sqrt{s} = 3.5$  TeV) with  $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$ . The cuts imposed are summarized in Sec. IIIB.

FIG. 15. Limit contours at the 95% CL for  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV derived from the  $p_T(Z)$  distribution. Contours are shown in three planes: a) the  $\Delta\kappa^0 - \lambda^0$  plane, b) the  $\Delta\kappa^0 - \Delta g_1^0$  plane, and c) the  $\Delta g_1^0 - \lambda^0$  plane. The solid and dashed lines give the inclusive NLO and LO results, respectively, for  $\int \mathcal{L} dt = 10 \text{ fb}^{-1}$ . The dotted and dot-dashed curves show the results obtained from the exclusive NLO  $WZ + 0$  jet channel for integrated luminosities of  $10 \text{ fb}^{-1}$  and  $100 \text{ fb}^{-1}$ , respectively. The cuts imposed are summarized in Sec. IIIB.

FIG. 16. Rapidity spectrum of the  $Z$  boson in the  $WZ$  rest frame in the Born approximation for a)  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 1.8$  TeV and b)  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV. The curves are for the SM with reconstructed  $WZ$  rest frame (solid lines) and non-standard  $WWZ$  couplings (dotted, dashed, and dot-dashed curves) as listed on the figure. The lower (upper) lines apply for positive (negative) anomalous couplings. The dash-double-dotted line shows the true SM  $|y^*(Z)|$  distribution. The cuts imposed are summarized in Sec. IIIB.

FIG. 17. SM rapidity difference distributions in the Born approximation for a)  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 1.8$  TeV and b)  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  at  $\sqrt{s} = 14$  TeV. The curves are for  $y(Z) - y(\ell_1^+)$  (solid lines) and  $y(\ell_2^-) - y(\ell_1^+)$  (dotted lines). The cuts imposed are summarized in Sec. IIIB.

FIG. 18. The differential cross section for the rapidity difference  $\Delta y(Z, \ell_1)$  for  $p\bar{p} \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  a) at  $\sqrt{s} = 1.8$  TeV and b) at  $\sqrt{s} = 3.5$  TeV. The solid and dot-dashed curves show the inclusive NLO and the LO SM prediction, respectively. The dashed and dotted lines give the results for  $\Delta\kappa^0 = +1$  and  $\Delta\kappa^0 = -1$ , respectively. The error bars associated with the solid curves indicate the expected statistical uncertainties for an integrated luminosity of  $10 \text{ fb}^{-1}$ . The cuts imposed are summarized in Sec. IIIB.

FIG. 19. The differential cross section for the rapidity difference  $\Delta y(Z, \ell_1)$  at  $\sqrt{s} = 14$  TeV for a)  $pp \rightarrow W^+Z + X \rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + X$  in the SM and b) for  $pp \rightarrow W^+Z + 0$  jet  $\rightarrow \ell_1^+\nu_1\ell_2^+\ell_2^- + 0$  jet at NLO. In part a) the dotted and dashed curves show the inclusive NLO and the LO SM prediction, respectively, while the solid curve gives the prediction for the SM NLO  $WZ + 0$  jet case. The error bars associated with the solid curve indicate the expected statistical uncertainties for an integrated luminosity of  $100 \text{ fb}^{-1}$ . In part b) the curves are for the SM (solid line),  $\Delta\kappa^0 = +1$  (dotted line),  $\lambda^0 = +0.25$  (dashed curve), and  $\Delta g_1^0 = +0.25$  (dot-dashed curve). The cuts imposed are summarized in Sec. IIIB. For the jet definition, we have used Eq. (14).

FIG. 20. The ratio  $\mathcal{R}_{ZZ/WZ} = B(Z \rightarrow \ell^+\ell^-)\sigma(ZZ)/B(W \rightarrow \ell\nu)\sigma(W^\pm Z)$ ,  $\ell = e, \mu$ , as a function of the minimum transverse momentum of the  $Z$  boson,  $p_T^{\min}$ , at a) the Tevatron and b) the LHC. The solid and dashed line show the inclusive NLO and the LO result for the SM, respectively. The dotted line in b) gives the SM cross section ratio at NLO if a 0-jet requirement is imposed. The dot-dashed line displays  $\mathcal{R}_{ZZ/WZ}$  for  $\Delta\kappa^0 = +1$ . In part a) this curve is calculated taking into account inclusive NLO QCD corrections, whereas in part b) the dot-dashed curve is for the NLO 0-jet cross section ratio. The cuts imposed are summarized in Sec. IIIB. For the jet definition, we have used Eq. (14).

FIG. 21. The ratio  $\mathcal{R}_{WZ/W\gamma} = B(Z \rightarrow \ell^+\ell^-)\sigma(W^\pm Z)/\sigma(W^\pm\gamma)$ ,  $\ell = e, \mu$ , as a function of the minimum transverse momentum of the  $Z$  boson and photon,  $p_T^{\min}$ , respectively, at a) the Tevatron and b) the LHC. The solid and dashed lines show the inclusive NLO and the LO result for the SM, respectively. The dotted and dot-dashed lines display the inclusive NLO and LO  $WZ$  to  $W\gamma$  cross section ratio for  $\Delta\kappa_\gamma^0 = \Delta\kappa^0 = -1$ . Here,  $\Delta\kappa_\gamma$  is the anomalous  $WW\gamma$  coupling defined in an analogous way to  $\Delta\kappa$  [see Eq. (1)]. The cuts imposed are summarized in Sec. IIIB and Eq. (20).

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